Network Medicine

Petri Nets: Extensions

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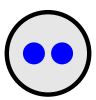
Petri Nets: Definition

$$N = (P, T, W, M_0)$$

- $P = \{p_1, \ldots, p_n\}$: the set of places 
- $T = \{t_1, \ldots, t_m\}$: the set of transitions 
- $W : \big((P \times T) \cup (T \times P)\big) \to \mathbb{N}$: the weight function
  - assigns multiplicities to arcs 

- $M_0 : P \to \mathbb{N}$: the initial marking
  - the initial number of tokens in places 

Petri Nets and Turing Completeness $^{1}/_{3}$

Are Petri nets Turing complete?

$=$

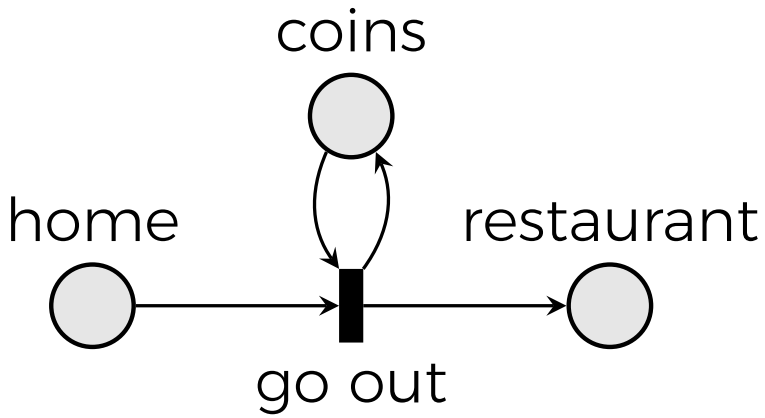
Can any algorithm be implemented?

$=$

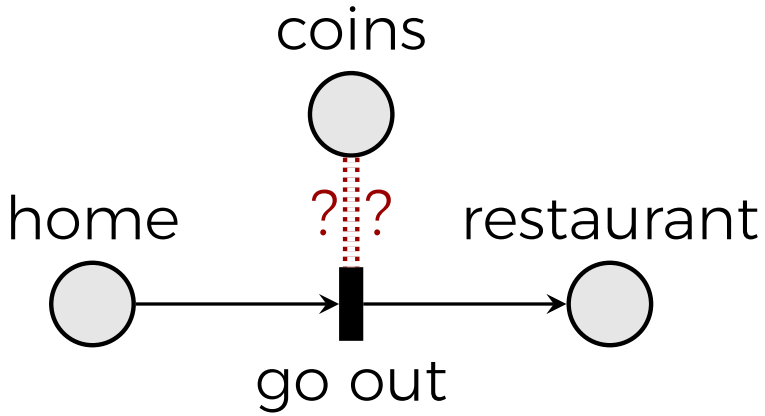
Can any system be modelled?

Petri Nets and Turing Completeness $^{2}/_{3}$

Model: "I only go out if I have money."



Model: "I don't go out if I have money."



Petri Nets and Turing Completeness $^{3}/_{3}$

Standard Petri nets are not Turing complete.

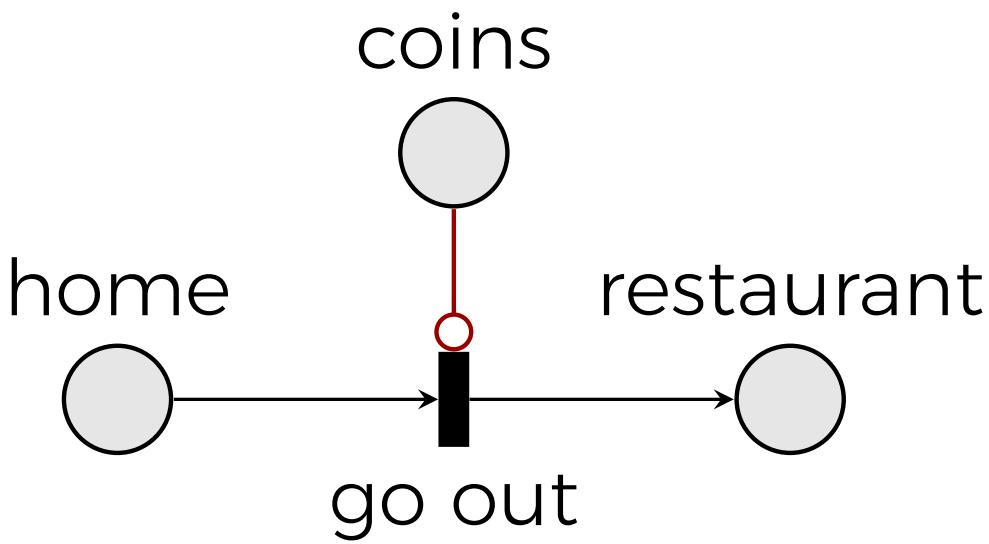
— not everything can be implemented/modelled

\+ automatic analysis is possible

- Turing-complete systems are difficult to analyse (too expressive)

Petri Nets with Inhibitor Arcs

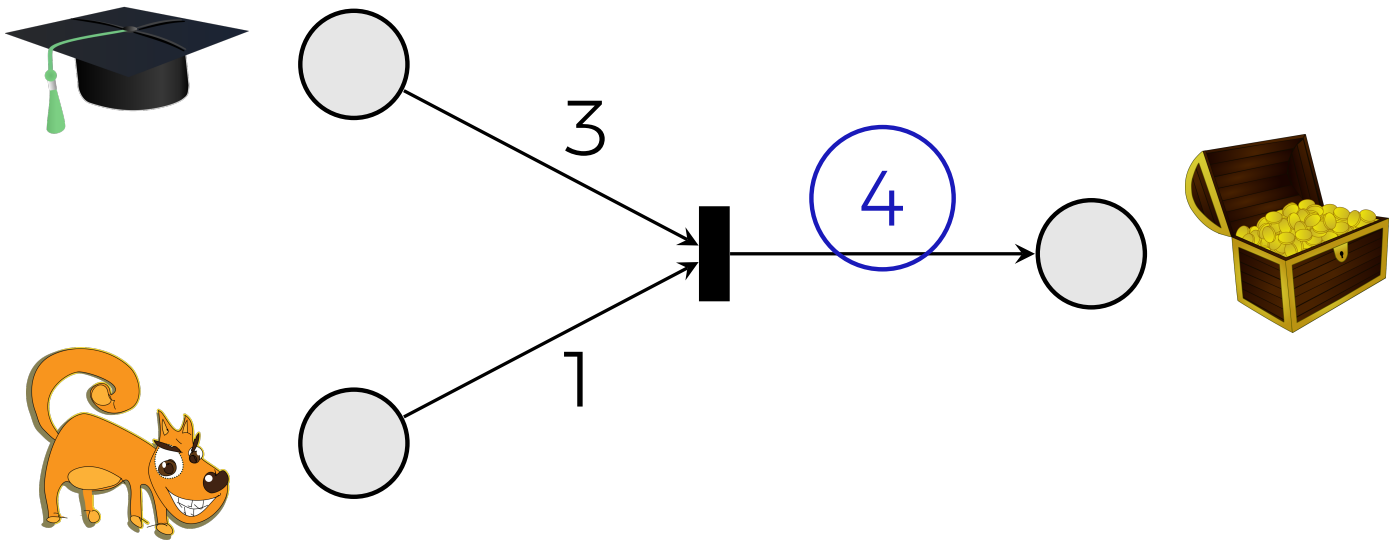
"I don't go out if I have money."



"go out" is inhibited by presence of tokens in "coins".

Petri nets with inhibitor arcs are Turing complete.

Token Discrimination



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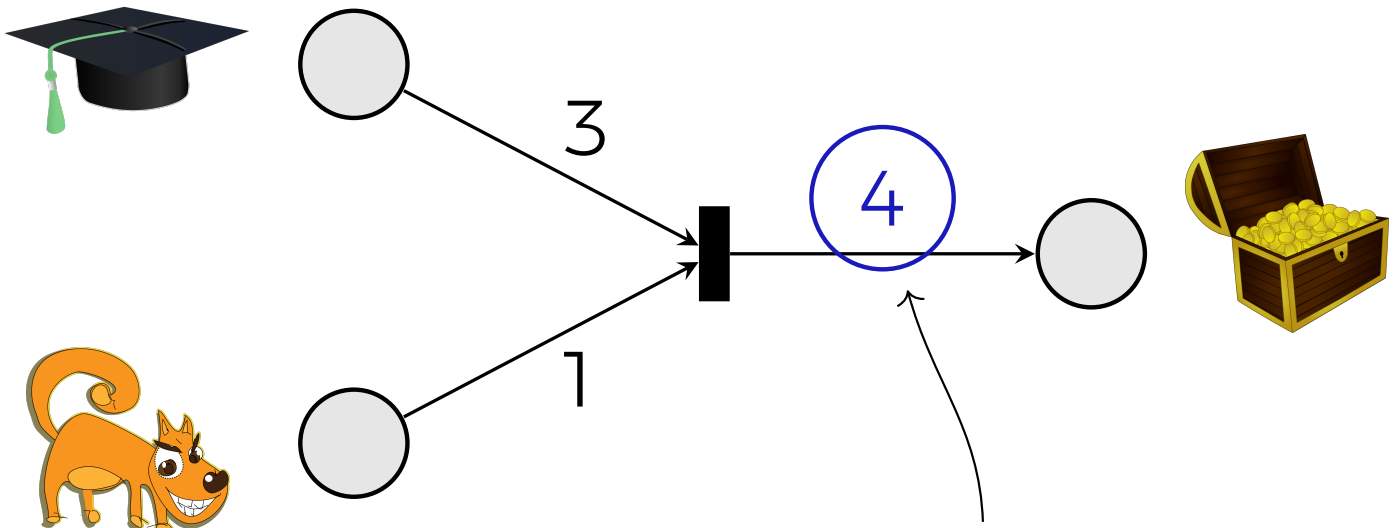
Token Discrimination



How to differentiate between the tokens?

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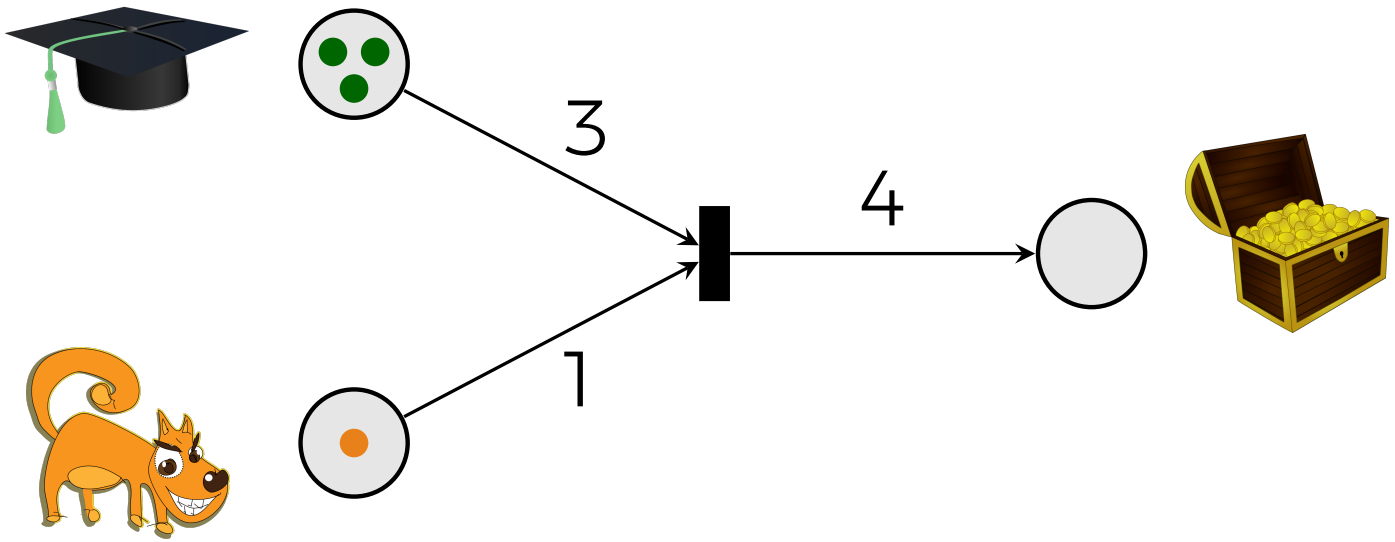
Coloured Petri Nets CPN

Tokens may be distinguished by colours.



Colours are assigned arbitrarily.

- no relationship with places is enforced

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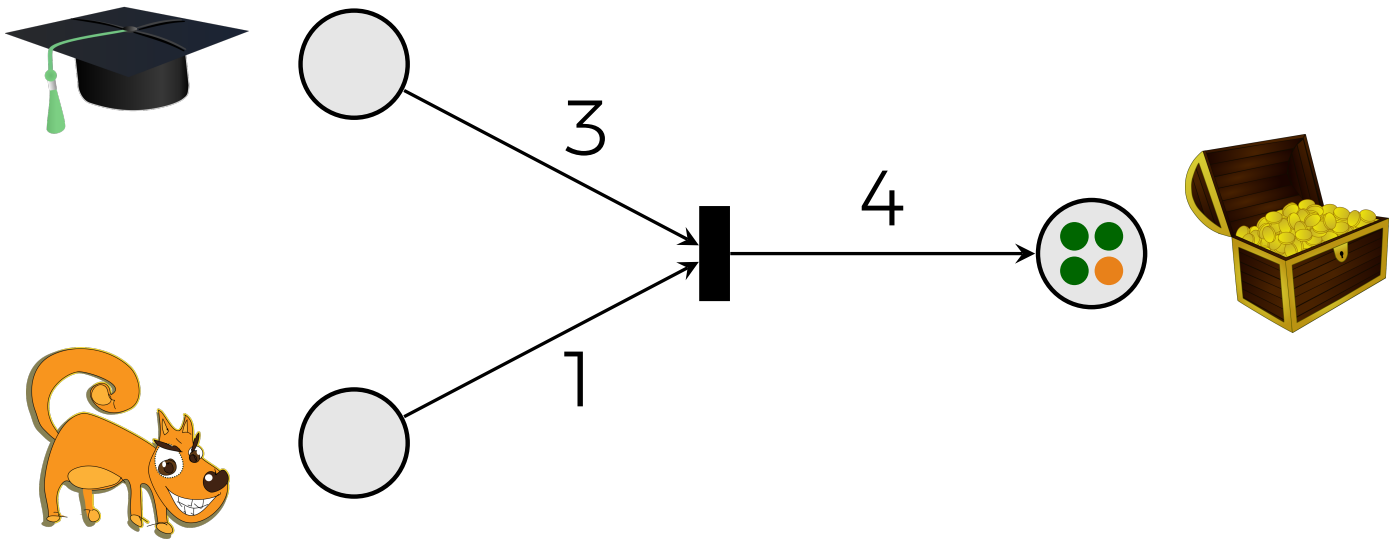

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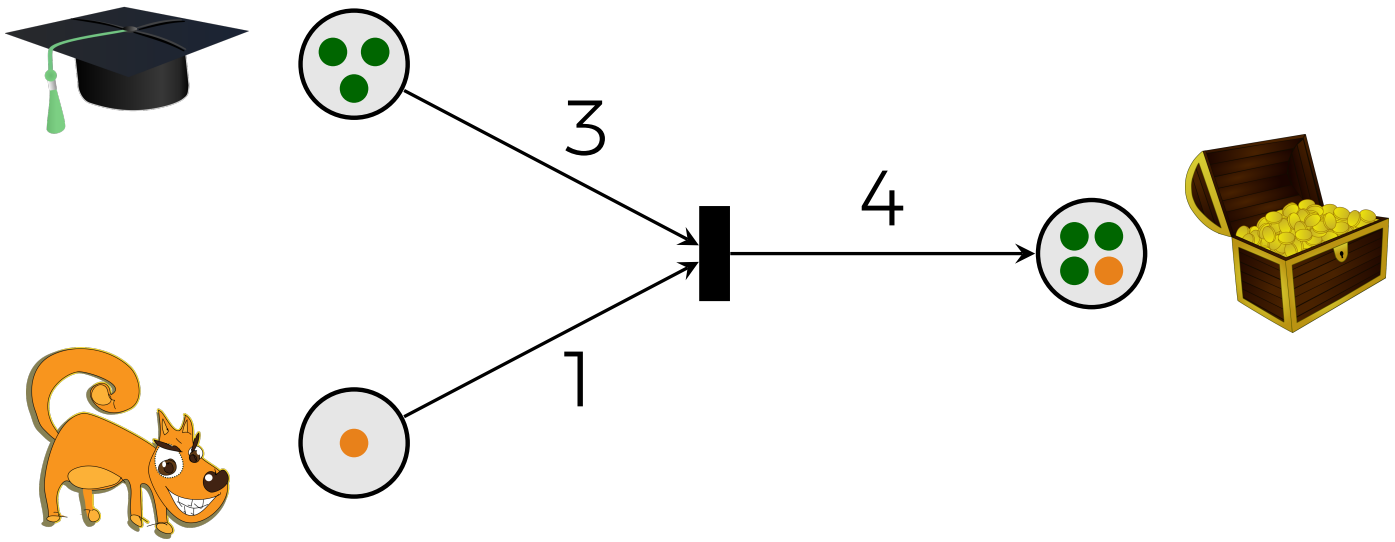

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Coloured Petri Nets and Turing Completeness

Are coloured Petri nets Turing complete?

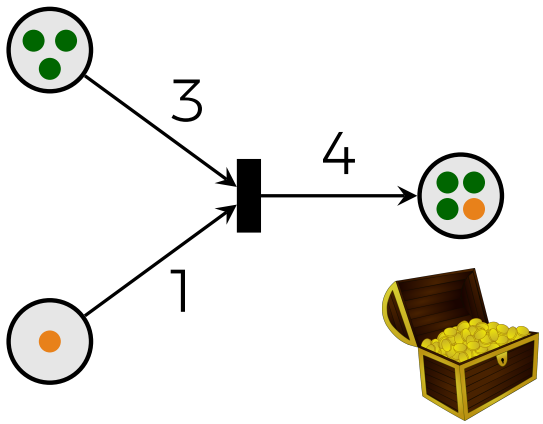
=

Can any algorithm be implemented?

=

Can any system be modelled?

Simple Coloured Petri Nets: Not Turing Complete



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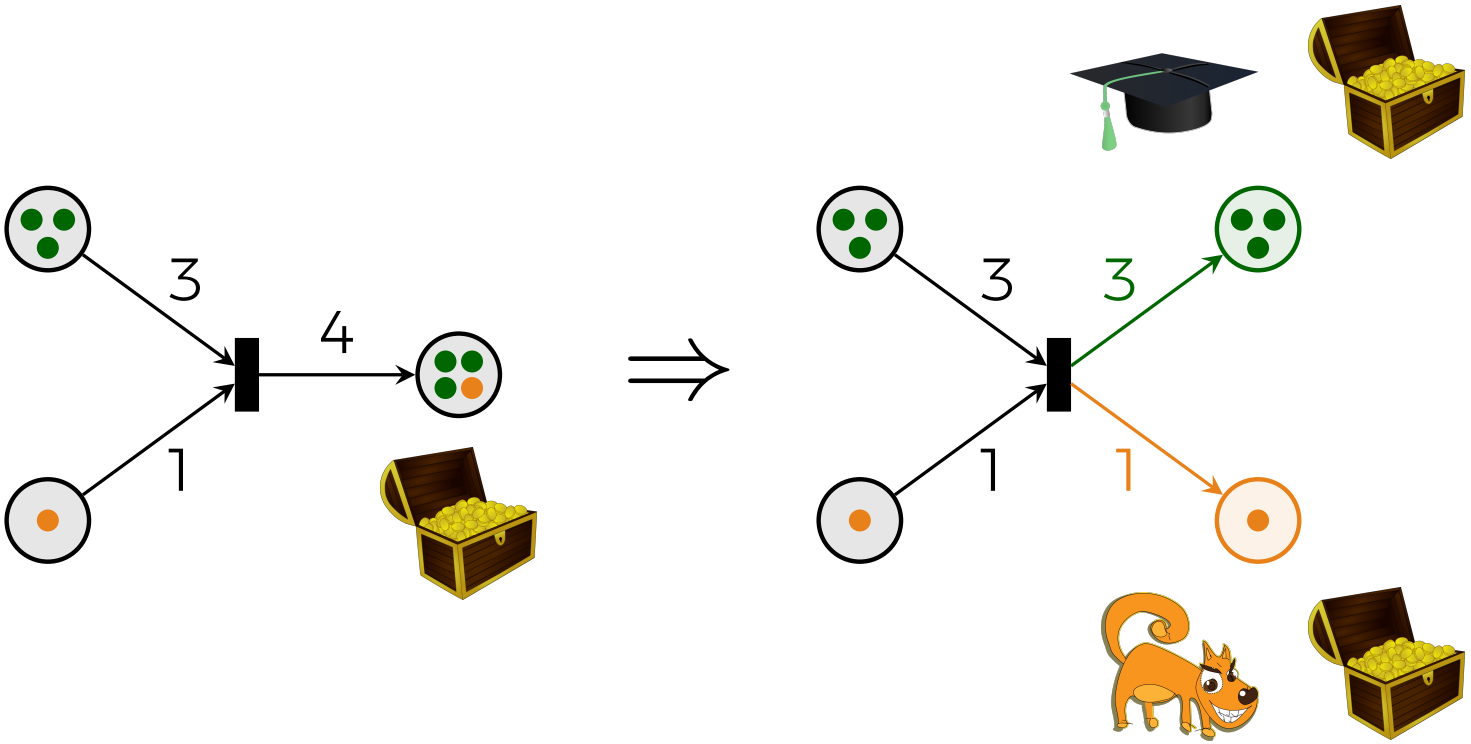
Simple Coloured Petri Nets: Not Turing Complete



Coloured tokens = multiple places

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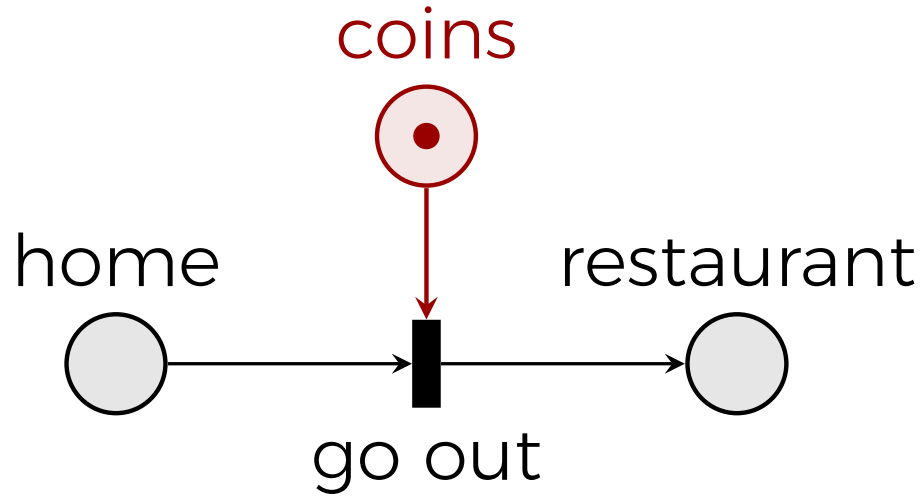
Coloured Petri Nets with Guards

Transitions in coloured Petri nets are typically equipped with guards.

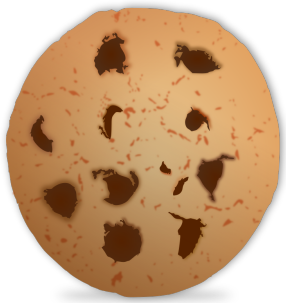


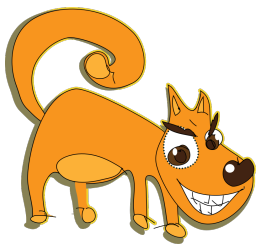
$$\text{guards}(\text{go out}) = \text{number}(\bullet) > 0$$

Coloured Petri nets with guards are Turing complete

- \+ expressive modelling
- − difficult analysis

Concurrency in Time





The dog may consume the cookie between 9am and 6pm.

I may consume the cookie between 7pm and 1am.

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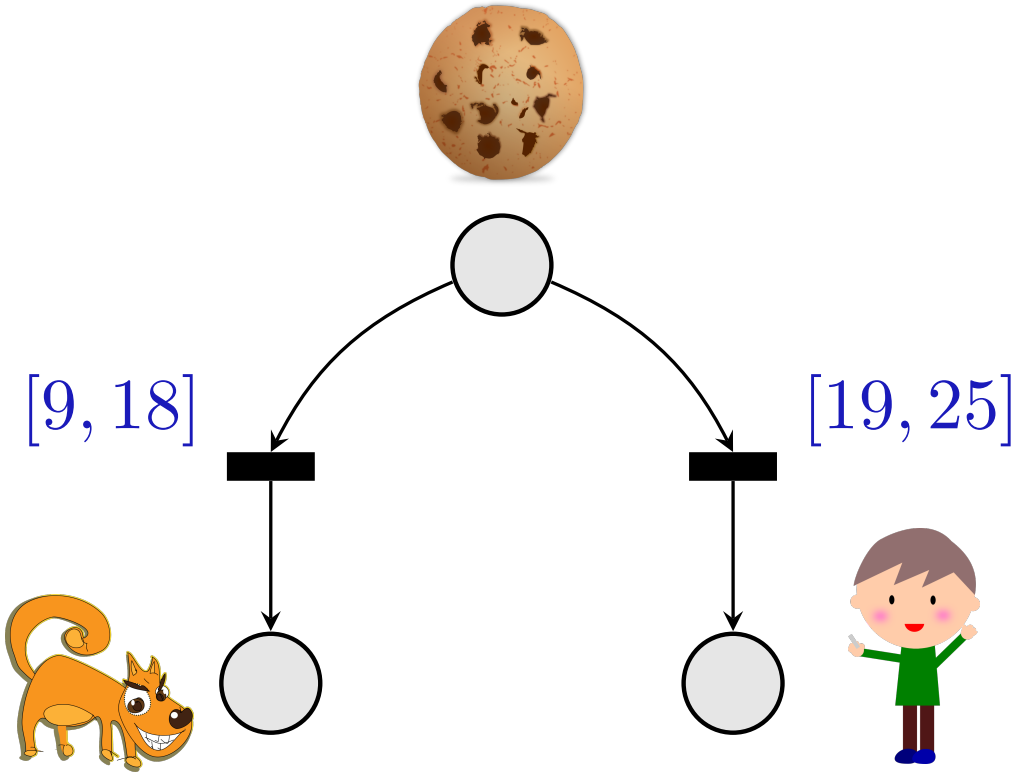
Timed Petri Nets



A time interval $[a_t, b_t]$ is associated with each transition t.

- t must fire after at least a_t time units
- t must fire before at most b_t time units

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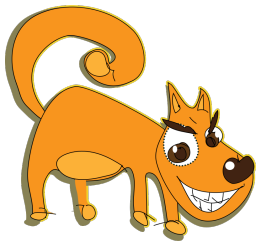
Randomised Concurrency



~~The dog may consume the cookie between 9am and 6pm.~~

~~I may consume the cookie between 7pm and 1am.~~

We just grab the cookie whenever we want!

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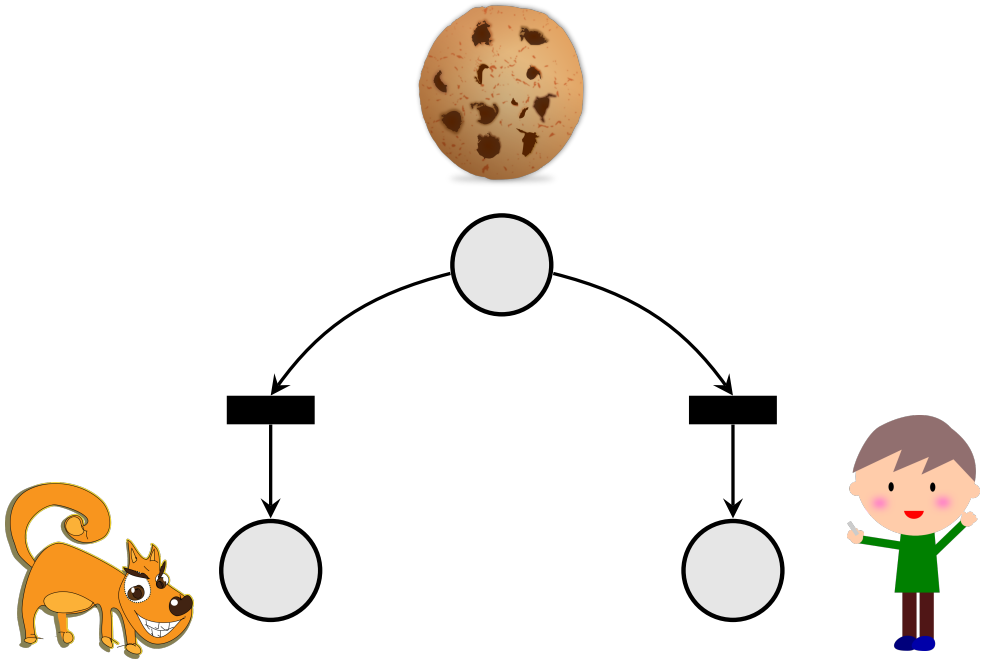
Stochastic Petri Nets



A random variable X_t is associated with each transition t.

The firing delay of t is given by X_t.

+ easier to analyse — continuous-time Markov chains
− less modelling expressiveness for time

Cookie Metabolism

Model the following:

- I have 1.5 cookies left.
- Eating 0.3 of a cookie makes me feel 20% better.
- I eat 0.5 cookies per hour.

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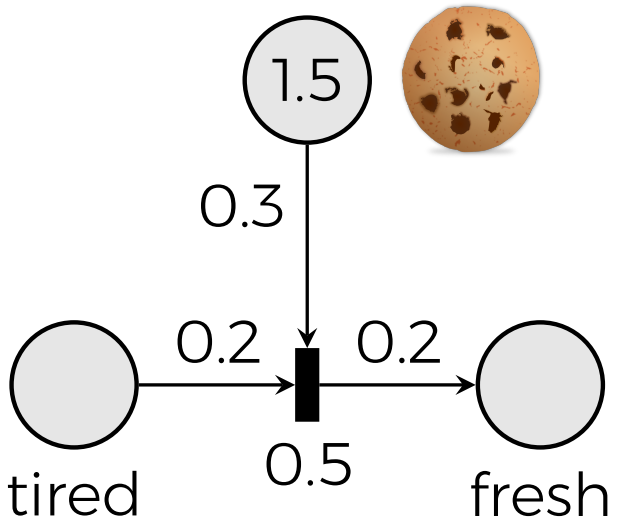
Conventional Petri nets are discrete—no fractional quantities or fractional consumption rates.

Continuous Petri Nets

- Places store continuous values
- Transitions have continuous firing rates
- Similar to ODE models



Convenient graphical notation for metabolic pathways.

Essentially a different formalism.

Summary

We have briefly discussed:

- Petri nets with inhibitor arcs
- coloured Petri nets CPN
- timed Petri nets
- stochastic Petri nets
- continuous Petri nets

There are more! We won't go there.

Baldan, P., Cocco, N., Marin, A. et al. "Petri nets for modelling metabolic pathways: a survey" Nat Comput (2010) 9: 955. https://doi.org/10.1007/s11047-010-9180-6