



Network Medicine

Petri Nets: Extensions

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Petri Nets: Definition

$$N = (P, T, W, M_0)$$

- ▶ $P = \{p_1, \dots, p_n\}$: the set of places
- ▶ $T = \{t_1, \dots, t_m\}$: the set of transitions
- ▶ $W: ((P \times T) \cup (T \times P)) \rightarrow \mathbb{N}$: the weight function
 - ▶ assigns multiplicities to arcs
- ▶ $M_0: P \rightarrow \mathbb{N}$: the initial marking
 - ▶ the initial number of tokens in places



Are Petri nets Turing complete?

=

Can any algorithm be implemented?

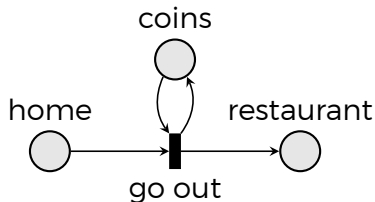
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Can any system be modelled?

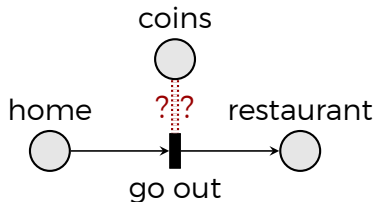
Petri Nets and Turing Completeness

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Model: "I only go out if I have money."



Model: "I don't go out if I **have** money."

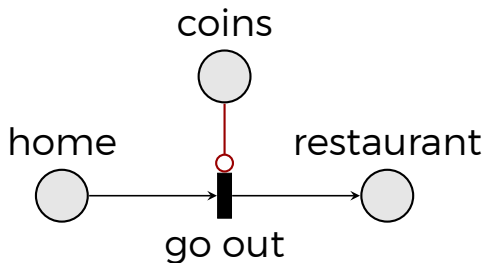


Standard Petri nets are **not** Turing complete.

- not everything can be implemented/modelled
- + automatic analysis is possible
 - ▶ Turing-complete systems are difficult to analyse (too expressive)

Petri Nets with Inhibitor Arcs

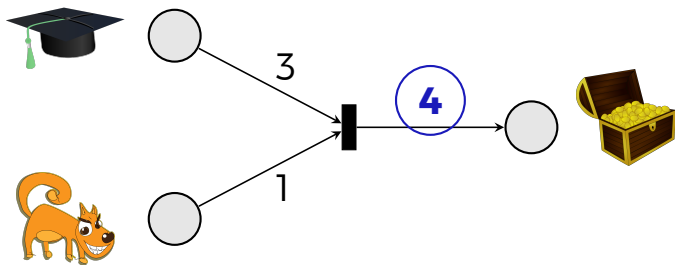
“I don't go out if I **have** money.”



“go out” is **inhibited** by presence of tokens in “coins”.

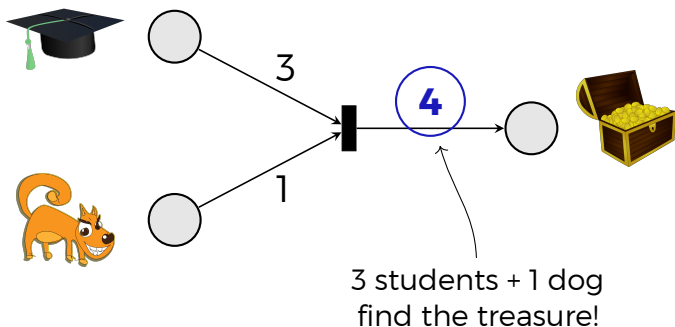
Petri nets with inhibitor arcs are **Turing complete**.

Token Discrimination



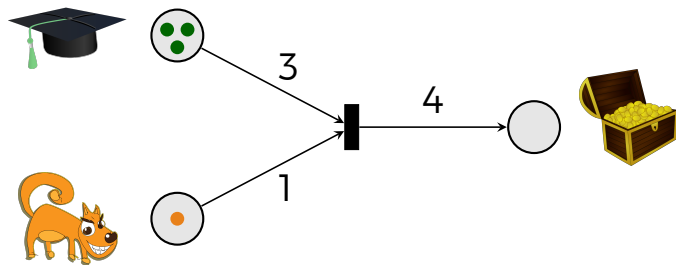
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Token Discrimination



How to **differentiate** between the **tokens**?

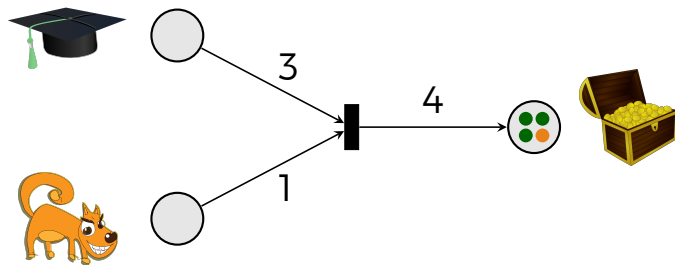
Tokens may be distinguished by **colours**.



Colours are assigned arbitrarily.

- ▶ no relationship with places is enforced

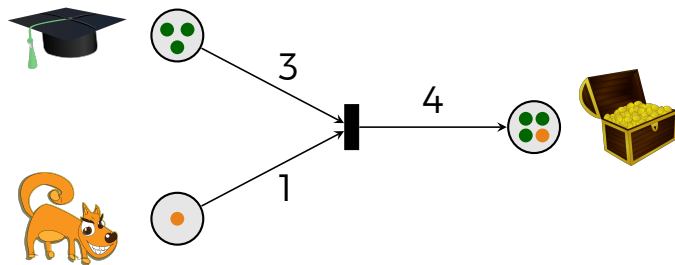
Tokens may be distinguished by colours.



Colours are assigned arbitrarily.

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Tokens may be distinguished by **colours**.



Colours are assigned arbitrarily.

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Coloured Petri Nets and Turing Completeness

Are **coloured** Petri nets **Turing complete**?

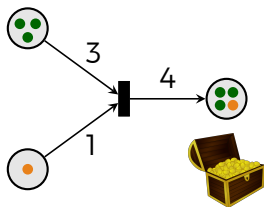
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Can **any algorithm** be implemented?

=

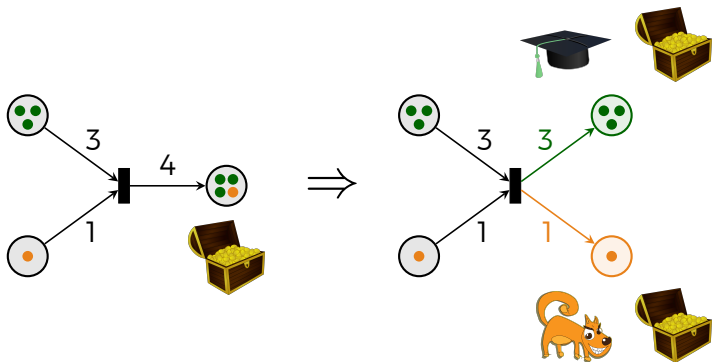
Can **any system** be modelled?

Simple Coloured Petri Nets: **Not** Turing Complete



<https://openclipart.org/>

Simple Coloured Petri Nets: **Not** Turing Complete



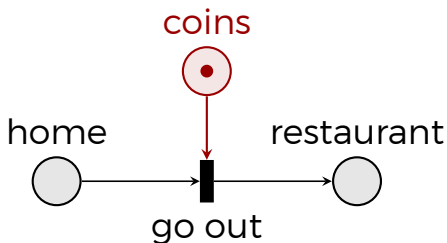
Coloured tokens = multiple places



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Coloured Petri Nets with Guards

Transitions in coloured Petri nets are typically equipped with guards.



$$\text{guards}(\text{go out}) = \text{number}(\bullet) > 0$$

Coloured Petri nets with **guards** are Turing complete

- + expressive modelling
- difficult analysis

Concurrency in Time



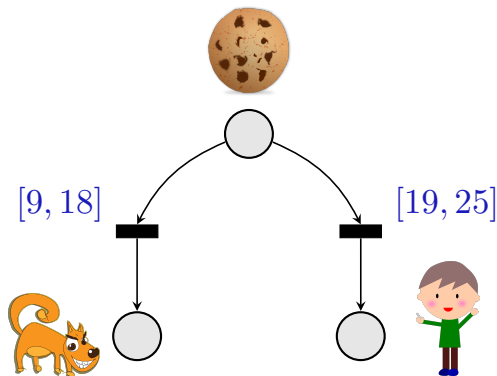
The dog may consume the cookie between 9am and 6pm.



I may consume the cookie between 7pm and 1am.

<https://openclipart.org/>

Timed Petri Nets



A time interval $[a_t, b_t]$ is associated with each transition t .

- ▶ t must fire **after** at least a_t time units
- ▶ t must fire **before** at most b_t time units

Randomised Concurrency



The dog may consume the cookie between 9am and 6pm.

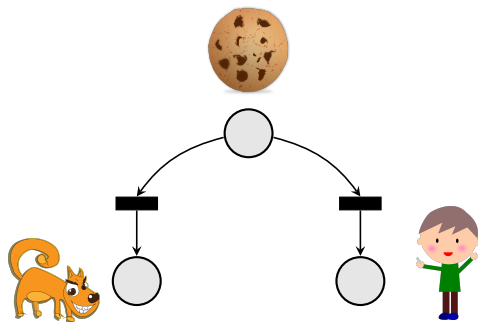
I may consume the cookie between 7pm and 1am.

We just grab the cookie whenever we want!



<https://openclipart.org/>

Stochastic Petri Nets



A random variable X_t is associated with each transition t .

The firing delay of t is given by X_t .

- + easier to analyse continuous-time Markov chains
- less modelling expressiveness for time

Cookie Metabolism

Model the following:

- ▶ I have 1.5 cookies left.
- ▶ Eating 0.3 of a cookie makes me feel 20% better.
- ▶ I eat 0.5 cookies per hour.

Cookie Metabolism

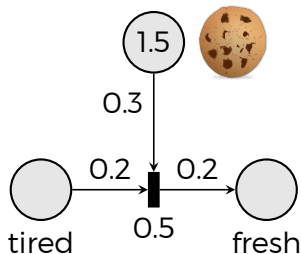
Model the following:

- ▶ I have 1.5 cookies left.
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-

Conventional Petri nets are discrete—no fractional quantities or fractional consumption rates.

Continuous Petri Nets

- ▶ Places store **continuous** values
- ▶ Transitions have **continuous** firing rates
- ▶ Similar to ODE models



Convenient graphical notation for **metabolic pathways**.


Essentially a **different** formalism.

Summary

We have **briefly** discussed:

- ▶ Petri nets with inhibitor arcs
- ▶ coloured Petri nets CPN
- ▶ timed Petri nets
- ▶ stochastic Petri nets
- ▶ continuous Petri nets

There are more! We won't go there.

 Baldan, P., Cocco, N., Marin, A. et al. "Petri nets for modelling metabolic pathways: a survey" Nat Comput (2010) 9: 955. <https://doi.org/10.1007/s11047-010-9180-6>