The Many Shapes of Polymorphism

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Artiom Alhazov, Rudolf Freund, Yurii Rogozhin,
Francis George Cabarle, The Sevillan Team, ...

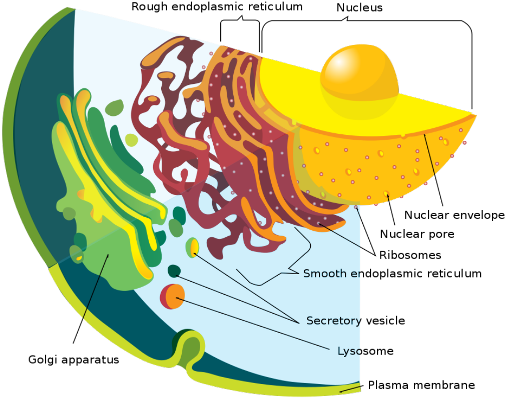
[E+A] CMC 2022

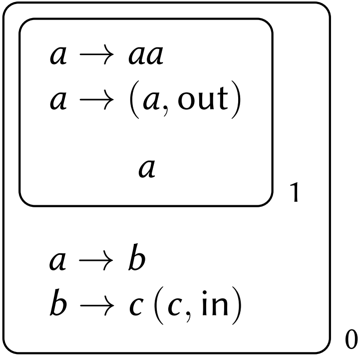
Artiom Alhazov, Rudolf Freund, Sergiu Ivanov:

Polymorphic P Systems: A Survey.

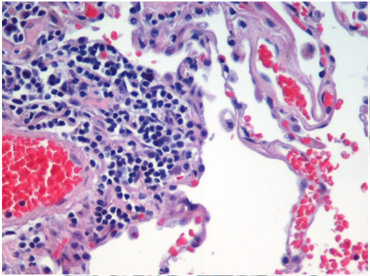
Bulletin of the International Membrane Computing Society (IMCS). Number 2, December 2016: 79–101.

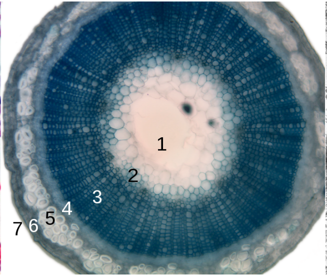
P systems are inspired by eukaryotes

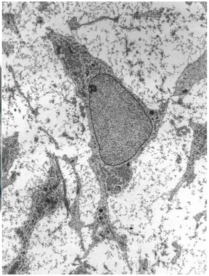


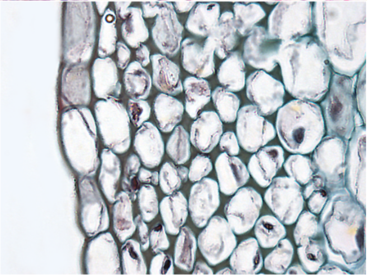


https://en.wikipedia.org/wiki/Eukaryote#Internal_membranes





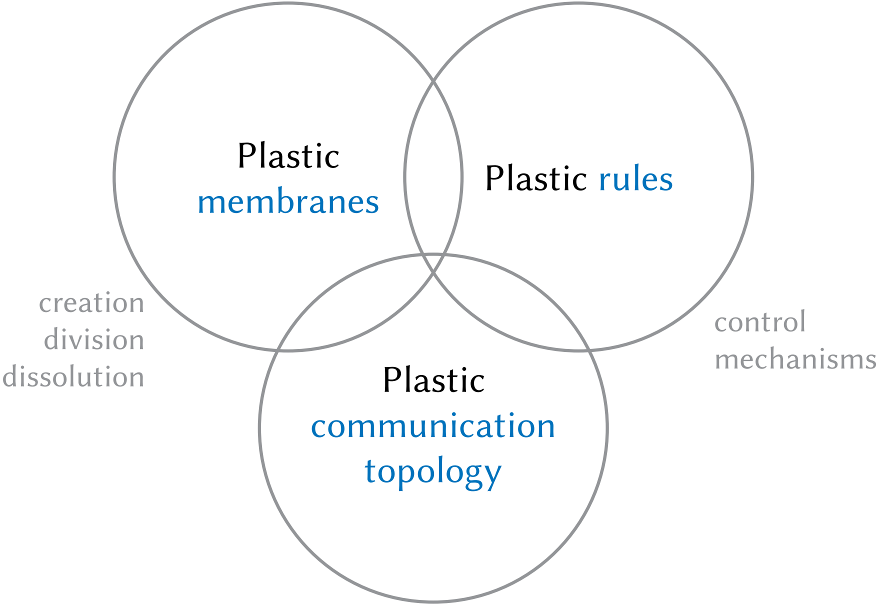




Life is plastic

https://en.wikipedia.org/wiki/File:Emphysema_H_and_E.jpg
https://en.wikipedia.org/wiki/File:Stem-histology-cross-section-tag.svg
https://en.wikipedia.org/wiki/File:MSC_high_magnification.jpg
https://en.wikipedia.org/wiki/File:Plant_cell_type_collenchyma.png

Plasticity in P systems



Time-varying

Randomized RHS

Control nuclei

States

Catalysts and activators

Polarizations

Active membranes

Promoters and inhibitors

Polymorphic P systems

Activation and blocking

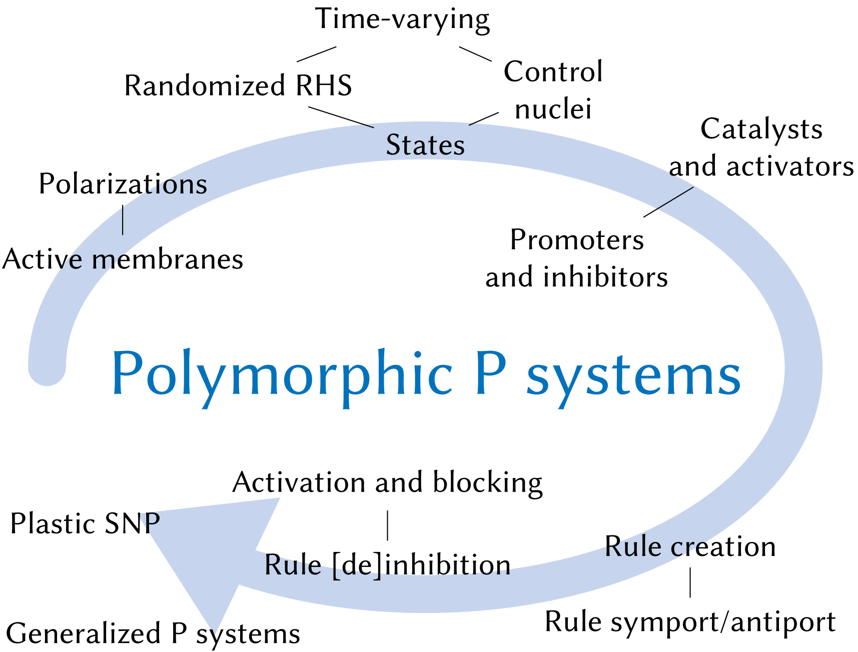
Plastic SNP

Rule [de]inhibition

Rule creation

Rule symport/antiport

Generalized P systems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Active membranes and polarizations

membrane division

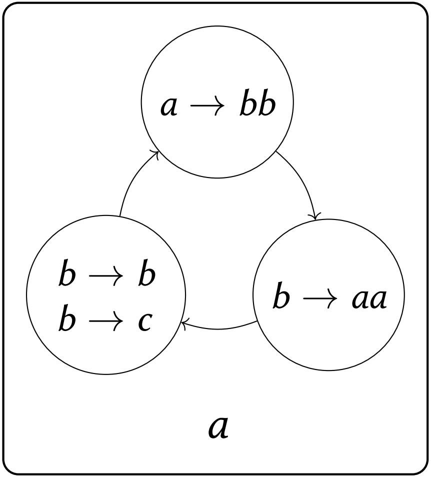
$$[a]_h \to [b]_{h'} \, [c]_{h''}$$

Replicate (or separate) membrane contents.
Replicate (or separate) computers.

$$[a]_h^+ \to [b]_h^-$$

Membrane polarization determines available rules.

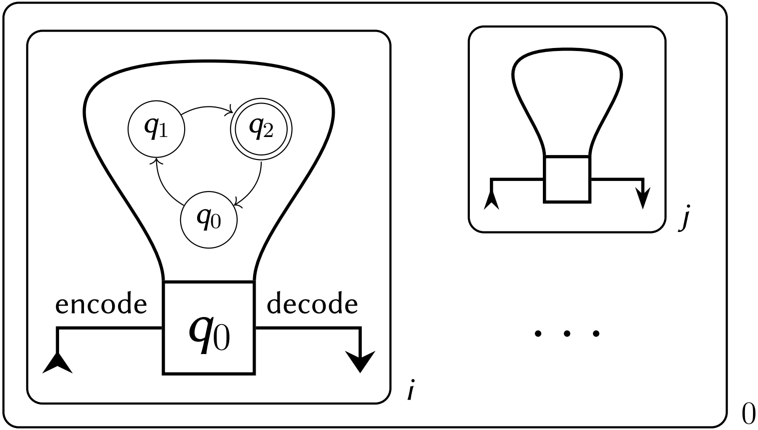
Time-varying P systems



The set of available rules varies cyclically with time.

Artiom Alhazov, Rudolf Freund, Hilbert Heikenwälder, Marion Oswald, Yurii Rogozhin, Sergey Verlan: Sequential P Systems with Regular Control. Int. Conf. on Membrane Computing 2012: 112-127

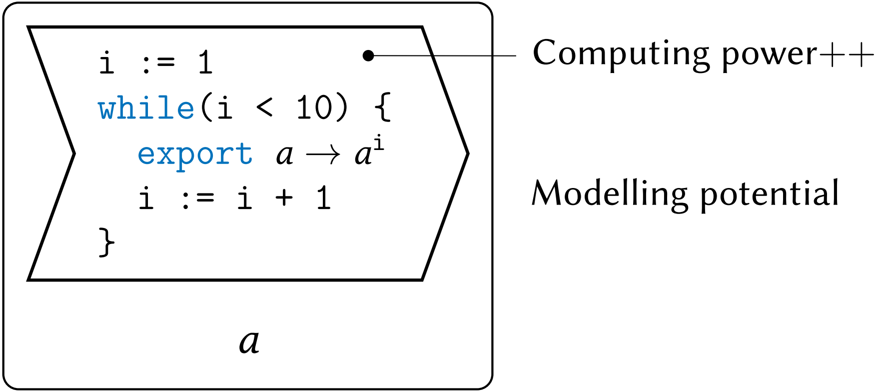
P systems with states



Potentially complex and infinite state transitions.

Artiom Alhazov, Rudolf Freund, Sergiu Ivanov, Marion Oswald: Observations on P Systems with States. Multidisciplinary Creativity, Marian Gheorghe, Ion Petre, Mario J. Pérez-Jiménez, Grzegorz Rozenberg, and Arto Salomaa, eds. p. 17-28, Editura Spandugino, 2015.

Control nuclei



Ştefănescu, G., Şerbănuţa, T., Chira, C., Roşu, G.: P Systems with Control Nuclei. In: Preproceedings of the Tenth Workshop on Membrane Computing (WMC10), Curtea de Argeş, 361–365 (2009)

Randomized right-hand sides

$a \to aa$

$c \to b$

a

Random permutation at every step.

- identity allowed
- increases the power

$$N = \{2^n \mid n \in \mathbb{N}\}$$

Artiom Alhazov, Rudolf Freund, Sergiu Ivanov: P systems with randomized right-hand sides of rules. Theor. Comput. Sci. 805: 144-160 (2020)

Promoters and inhibitors

$$a \to aa \mid_b$$
$$b \to b$$
$$b \to \lambda$$
$$a$$

$$a \to aa \mid_{\neg d}$$
$$a \to b \mid_{\neg c}$$
$$c \to c$$
$$c \to d$$
$$d \to \lambda$$
$$ca$$

$$N = \{2^n \mid n \in \mathbb{N}\}$$

Catalytic P systems

Control the parallelism

$$a \to aa \,\big|_p \qquad\qquad ca \to caa$$

multiply by 2 — increment

Purely catalytic: bound the parallelism

- all rules are catalytic

P systems with activators

$$r_1 : (a : b \to aa)$$
$$r_2 : (b : a \to \lambda)$$

$$a^3 b^5$$

- a activates r_1 3 times
- a evolves by r_2
- b activates r_2 5 times
- b evolves by r_1

Artiom Alhazov: A Note on P Systems with Activators. In: Gh. Păun, A. Riscos-Núñez, A. Romero-Jiménez, F. Sancho-Caparrini: RGNC report 01/2004, Second Brainstorming Week on Membrane Computing, Sevilla, 2004, 16-19.

Rule creation

$$r : a \to w/z, \;\; z \in RuleLabels^*$$

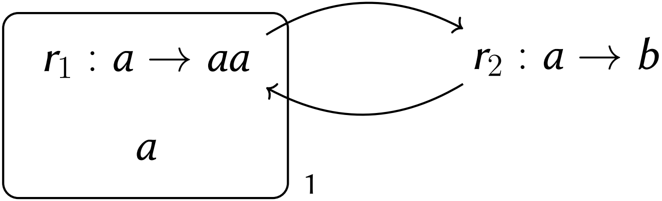
1. consume applied rules
2. reintroduce the rules in z

- Rule reintroduction rather than rule creation
- Similar to time-varying

Fernando Arroyo, Angel Baranda, Juan Castellanos, Gheorghe Păun: Membrane Computing: The Power of (Rule) Creation. Journal of Universal Computer Science, vol. 8, no. 3 (2002), 369-381

Symport/antiport of rules

$$(r_2, in; r_1, out)$$



$$N = \{2^n \mid n \in \mathbb{N}\}$$

Cavaliere, M., Genova, D.: P systems with Symport/Antiport of Rules. Journal of Universal Computer Science, vol. 10, 5, 540–558 (2004)

Inhibiting/de-inhibiting rules

$r : a \to w \langle r_1 \dots r_n \rangle$ toggle the inhibition of $r_1 \dots r_n$

$r_1 : a \to aa$ $r_2 : a \to aa \langle r_1 r_2 r_3 \rangle$

$r_3 : a \to b$ a

1

starts inhibited

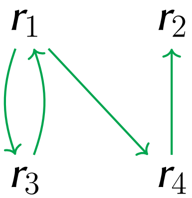
$$N = \{2^n \mid n \in \mathbb{N}\}$$

Matteo Cavaliere, Mihai Ionescu, Tseren-Onolt Ishdorj: Inhibiting/ De-inhibiting Rules in P Systems. Workshop on Membrane Computing 2004: 224-238

Activation/Blocking of rules

Activation graph

$$r_1 : a \to aa \qquad r_2 : a \to b$$
$$r_3 : c \to c \qquad r_4 : c \to d$$
$$ca$$
$$1$$



$$N = \{2^n \mid n \in \mathbb{N}\}$$

$$\ldots + \text{blocking} + \text{delays} \leq 2 = \text{PsRE}$$

Artiom Alhazov, Rudolf Freund, Sergiu Ivanov: Variants of P systems with activation and blocking of rules. Nat. Comput. 18(3): 593-608 (2019)

Generalized P systems

P systems with operators:

- control symbols
- rewriting
- object transfer operators
- operator transfer operators

operatorception

A rule: $(\underbrace{op_1, \ldots, op_k}_{\text{operators acting on objects}} ; \underbrace{op'_1, \ldots, op'_k}_{\text{operators acting on operators}})$

operators acting on objects

operators acting on operators

Rudolf Freund: Generalized P-Systems with Splicing and Cutting/Recombination. Grammars 2(3): 189-199 (1999)

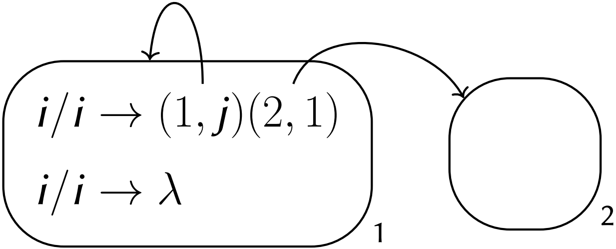
Extended Spiking Neural P Systems

ESNP

Generate any semilinear language in a^*:

$A_i \to A_j\, a$

$A_i \to \lambda$



Artiom Alhazov, Rudolf Freund, Marion Oswald, Marija Slavkovik: Extended Spiking Neural P Systems. Workshop on Membrane Computing 2006: 123-134

ESNP with structural plasticity

simplified

$$\sigma_i : E/a^c \to \alpha N$$

$\alpha = +$ $\Longrightarrow$ connect σ_i to the neurons in N.

$\alpha = -$ $\Longrightarrow$ disconnect σ_i from the neurons in N.

$+$ usual ESNP rules

Francis George C. Cabarle, Henry N. Adorna, Mario J. Pérez-Jiménez, Tao Song: Spiking neural P systems with structural plasticity. Neural Comput. Appl. 26(8): 1905-1917 (2015)

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Generalized P systems

Code is data.

— John von Neumann

https://en.wikipedia.org/wiki/John_von_Neumann

Polymorphic rules

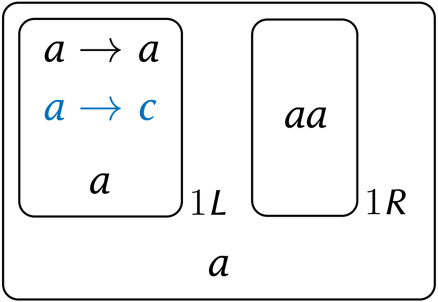
A rule	$r_1 : ab \rightarrow cd$
$\Downarrow$	$\Downarrow$
A pair of membranes	$[\,ab\,]_{1L}\quad[\,cd\,]_{1R}$

Rule LHS and RHS become membrane contents.

Outline

Generate non-semilinear languages

with halting



Graphical convention:

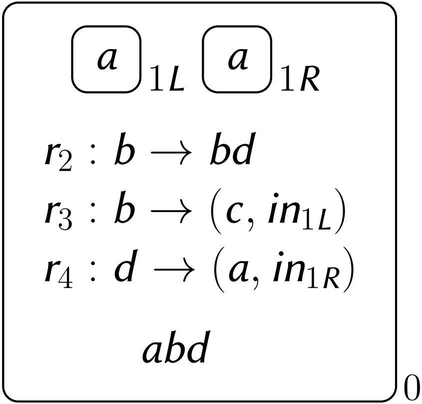
a c drawn as $a \to c$.

$a \to c$ switches off rule 1.

Nested rules!

$$N = \{2^n \mid n \in \mathbb{N}\}$$

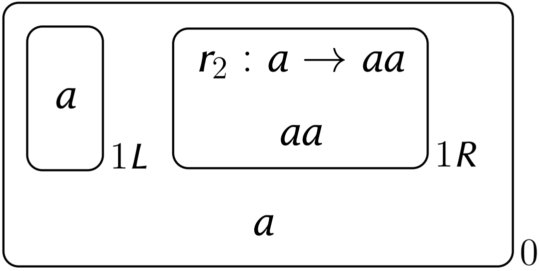
$n!$ in $O(n)$ steps



n	$\lvert 1R \rvert_a$	$\lvert 0 \rvert_a$
0	1	1
1	2	1
2	3	2
3	4	6
4	5	24
k	$k+1$	$k!$

- r_2 continues the multiplication
- r_3 disables $r_1 : (1L, 1R)$
- r_4 increments the factor
- no nested rules!
- target indications

Superexponential growth



n	$\lvert 1R\rvert_a$	$\lvert 0\rvert_a$
0	2	1
1	4	2
2	8	8
3	16	64
4	32	1024
k	2^{k+1}	$2^{\frac{k(k+1)}{2}}$

$$\lvert 0\rvert_a = 2^0 \times 2^1 \times 2^2 \times \cdots \times 2^k = 2^{\frac{k(k+1)}{2}}$$

Outline

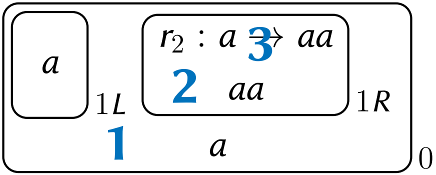
1. The power of polymorphism: first glance
1. The power of polymorphism: second glance
1. The power of polymorphism: third glance
2. Discussion

The growth rate

no target indications

$$I\, c^{p(n)}$$

- I : the size of the initial configuration
- c : max RHS of invariable rules
- $p(n)$: a polynomial of a degree $\leq$ depth $- 1$



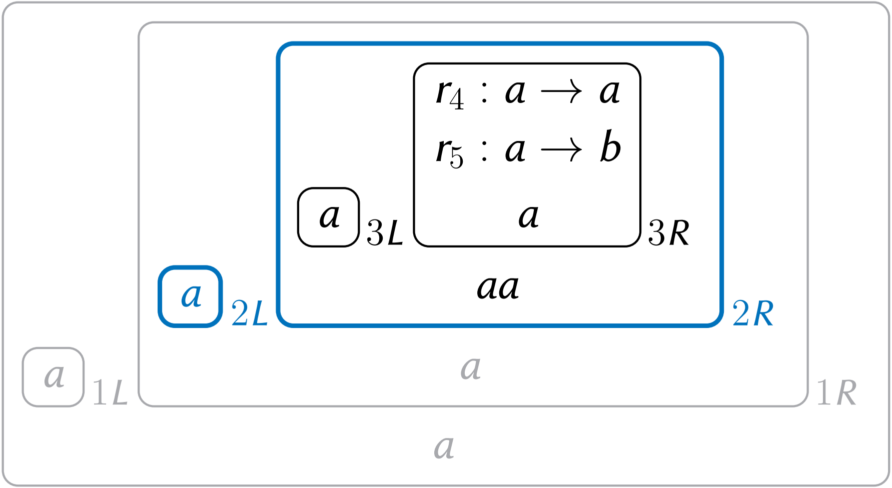
depth: 3

growth: $2^{\frac{n(n+1)}{2}}$

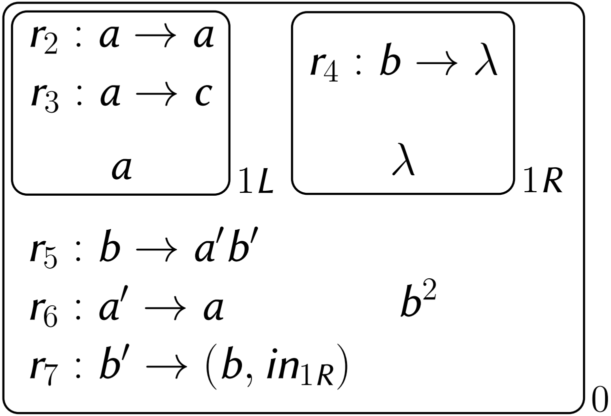
Sergiu Ivanov: Polymorphic P Systems with Non-cooperative Rules and No Ingredients. Int. Conf. on Membrane Computing 2014: 258-273

Generate superexponentials with halting

Make the RHS of $r_2 : a \to a^2$ dynamic.

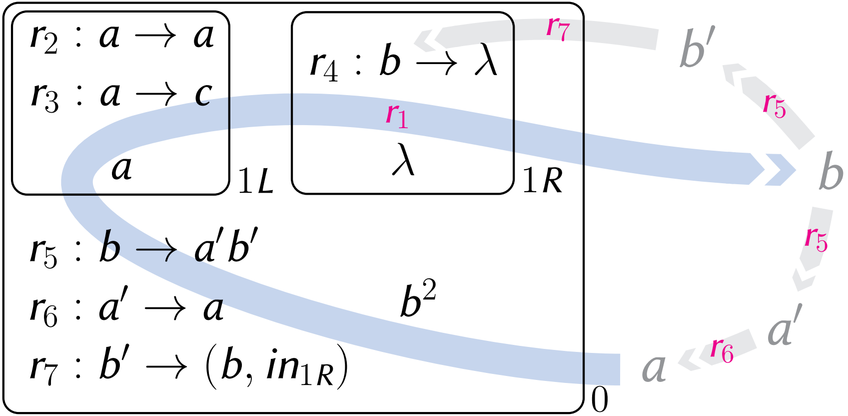


Superexponentials & target indications



n	0	1	2	3	4	5	6	$\cdots$	$3k$
$1R$	λ	λ	b^2	λ	λ	b^4	λ	$\cdots$	$2^{2^{k-1}}$
0	b^2	$(a'b')^2$	a^2	b^4	$(a'b')^4$	a^4	b^{16}	$\cdots$	2^{2^k}

Superexponentials & target indications



n	0	1	2	3	4	5	6	$\cdots$	$3k$
$1R$	λ	λ	b^2	λ	λ	b^4	λ	$\cdots$	$2^{2^{k-1}}$
0	b^2	$(a'b')^2$	a^2	b^4	$(a'b')^4$	a^4	b^{16}	$\cdots$	2^{2^k}

Outline

1. The power of polymorphism: first glance
2. The power of polymorphism: second glance
1. The power of polymorphism: third glance
1. Discussion

Establish sharper results

Weak & strong non-cooperativity

$$\boxed{u}_{iL}\ \boxed{v}_{iR}$$

Strong: $|u| \leq 1$ at any moment.

Weak: $|u| = 1$ whenever rule i is applied.

Convention: $u = \lambda \implies$ rule i is disabled.

Left & right polymorphism

$$\boxed{u}_{iL}\ \boxed{v}_{iR}$$

Left polymorphism: $v = const, \quad \forall i$

Right polymorphism: $u = const, \quad \forall i$

General polymorphism otherwise.

Notation

lpolym	left polymorphic
rpolym	right polymorphic
polym	general polymorphic

maximal depth
$*$ if unlimited

target indications

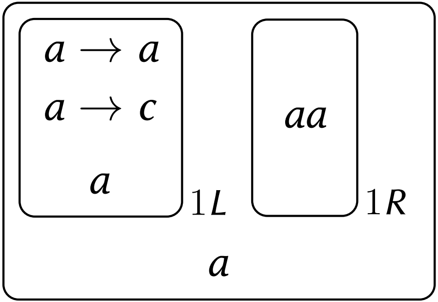
$$OP^h(polym_{+d}(ncoo_s, tar))$$

rule disabling

$ncoo_s$	strong non-cooperativity
$ncoo_w$	weak non-cooperativity

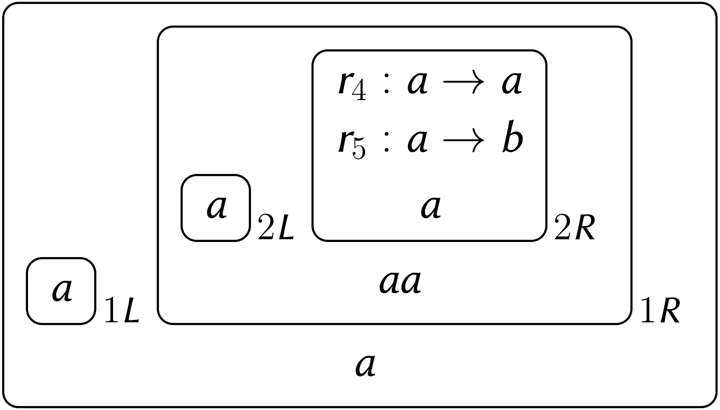
Left polymorphism > semilinear

$$\{2^n \mid n \in \mathbb{N}\} \in NOP^*(lpolym(ncoo))$$



Right polymorphism > semilinear

$$\{2^n \mid n \in \mathbb{N}, n > 2\} \in NOP^*(rpolym(ncoo))$$



The power of strong

$$PsOP^*(polym_{+d}(ncoo_s)) = PsOP^*(polym_{+d}(ncoo_w))$$

$$\boxed{u}_{iL}\ \boxed{v}_{iR}$$

1. *iL*: only non-cooperative rules $\implies$ regular behavior.
2. Replace by a finite automaton $\implies$ $|u| \leq 1$.

$+$ details

Corollary: Shallow left-hand sides

$$\mathrm{depth}(iL) \leq 2, \qquad \forall i, \textit{ncoo}, \cancel{\textit{tar}}$$

$$\boxed{u}_{iL} \; \boxed{v}_{iR}$$

1. iL is constant $\implies \mathrm{depth}(iL) = 1.$
2. iL is variable

   $\implies$ replace with a finite automaton

   $\implies \mathrm{depth}(iL) = 2.$

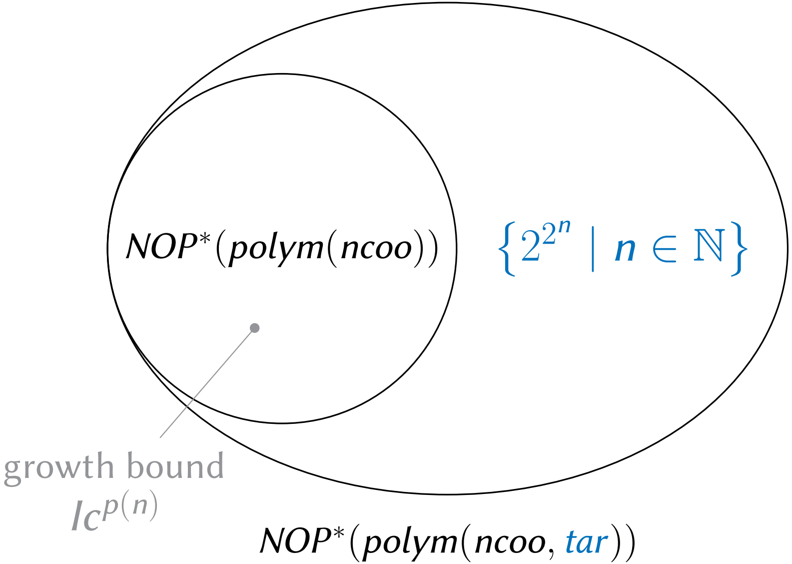
Corollary: No rule disabling

$$PsOP^*(polym_{-d}(ncoo)) = PsOP^*(polym_{+d}(ncoo))$$

$$\boxed{u}_{iL}\ \boxed{v}_{iR}$$

Replace λ in u with a fresh symbol.

The power of target indications



Targets separate $ncoo_s$ and $ncoo_w$?

Known: $\{n! \mid n \in \mathbb{N}\} \in NOP^*(polym(ncoo_{w}, tar))$.

Membrane 0 contains membranes $1L$ and $1R$ (each holding a), the rules

$r_2 : b \to bd$

$r_3 : b \to (c, in_{1L})$

$r_4 : d \to (a, in_{1R})$

and the objects abd.

$r_3 \implies r_1$ is $ncoo_{w}$.

Conjecture

$\{n! \mid n \in \mathbb{N}\} \notin NOP^*(polym(ncoo_{s}, tar))$.

An infinite hierarchy

h

. . .

$$NOP^{h+1}(polym(ncoo))$$

$$\left\{ 2^{\binom{n}{h-2}} \;\middle|\; n \in \mathbb{N}, n > h \right\}$$

$$NOP^{h}(polym(ncoo))$$

. . .

Open questions

Artiom Alhazov, Rudolf Freund, Sergiu Ivanov: Polymorphic P Systems: A Survey. Bulletin of the International Membrane Computing Society (IMCS). Number 2, December 2016: 79–101.

Computational power

1. $NOP^*(rpolym(ncoo)) \overset{?}{\subsetneq} NOP^*(polym(ncoo))$

   Easy: $NOP^h(lpolym(ncoo)) \subsetneq NOP^h(polym(ncoo))$

   - growth rate

2. Upper bounds on *polym*, *lpolym*, *rpolym*?

3. Better characterize target indications.

Better target indications

4. Polymorphism + per-symbol target indications?

5. Polymorphic targets?

6. Polymorphic tissue P systems?

Dissolution and division

7. Define dissolution
   - dissolve membrane containing polymorphic rules
   - dissolve rule sides?
8. Define division
   - divide rule sides?

Further ingredients?

Only target indications have been considered so far.

- polarizations?
- rule symport/antiport?
- $\cdots$

Applications

9. Is polymorphism easy to simulate? how?

10. Solve complex problems faster?

11. What type of target applications?

Back to plasticity

The meaning of plasticity

Everything counts?.. Meh!

The Turing Machine™: doing everything since 1936.

A P system variant → Computational power
a language easy!

A P system variant → Expressive power
 better capture intuitions

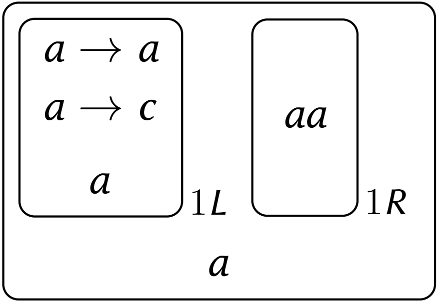
Plasticity

Plasticity | Polymorphism | Power

rpolym vs *polym*? upper bounds?

dissolution? power of targets?

division? finer targets?



applications? further ingredients?

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