Theory of Computer Science: Why All That Formal Stuff?

Sergiu Ivanov

sergiu.ivanov@univ-grenoble-alpes.fr

Université Grenoble Alpes

Open Seminar

Question

Computer Science $\leftrightarrow$ Maths

What is the relationship?

Outline

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Calculus

derivatives $\frac{df}{dx}$ integrals $\int_a^b f\,dx$

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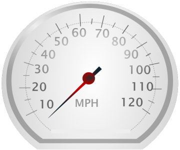

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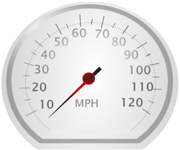

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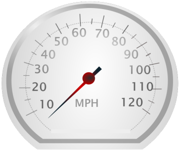
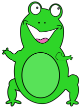


How often do we use that in practice?

We use that in games!



collisions, ray tracing, ...

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Outline

Formal Languages

Finite alphabet: $V = \{a_1, a_2, \ldots, a_n\}$ ← letters

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regular languages, finite automata, pushdown automata, Turing machines, context-free language, pumping lemma, ...

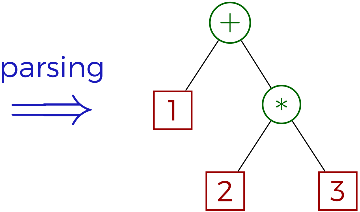
Why? Why?

Formal Languages: Compilers

Programming languages are formal languages

- alphabet for C $= \{\texttt{if}, \texttt{for}, \texttt{int}, \texttt{+}, \texttt{*}, \ldots\}$

$1 + 2 * 3$

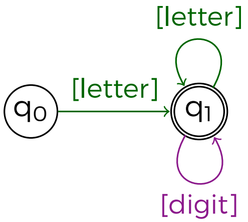


Compiler $=$ parser $+$ binary code generator

Formal Languages: Regular Expressions

[letter]([letter] | [digit])*

- a, ab, c2, x2a, ...



finite automaton

Formal regular expressions $\sim$ finite automata

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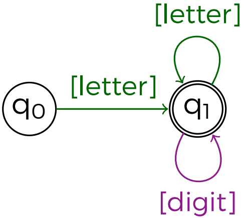

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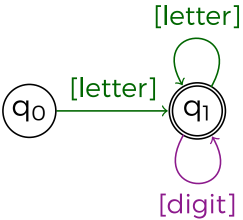
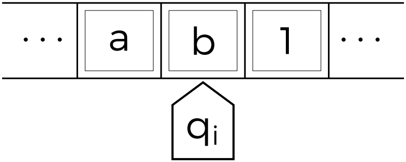
Regexp $=$ rather extended regular expressions

Formal Languages: A Philosophy of Computers

Finite automata



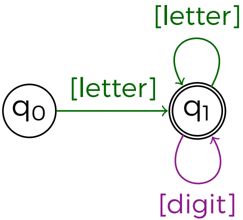
Turing machines

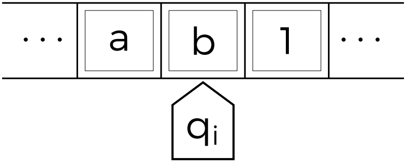


Formal Languages: A Philosophy of Computers

strictly less powerful

Finite automata $<$ Turing machines





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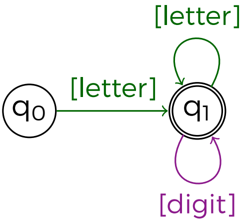

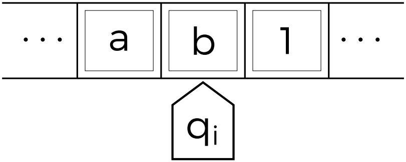

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strictly less powerful

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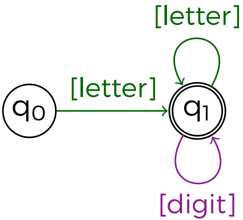

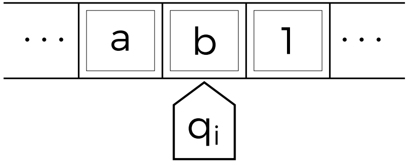

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Finite automata!

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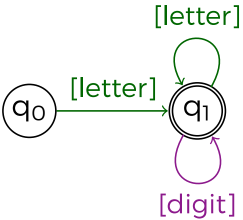

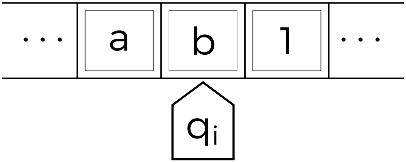

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but

Programming languages are Turing powerful!

Outline

Set Theory

$$A = \{a, b, c, \dots\}$$

When do programmers use set theory?

Classes and Types “are” Sets

A class/type is a set of objects sharing a property.

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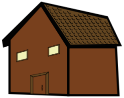
house = {  ,  ,  , . . . }

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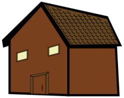
house = {  ,  ,  , . . . }

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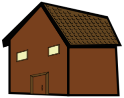
house = {  ,  ,  , . . . }

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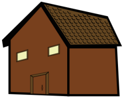
house = {  ,  ,  , . . . }

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Every house is a building, but not every building is a house.

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Types and Operations on Sets

`MyType x;` $\quad$ $x \in \text{MyType}$

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struct Person {
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How about $\cap$, $\setminus$, ... ?

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Outline

Abstract Algebra

Group 

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Abstract Algebra

Group

- associativity: $x + (y + z) = (x + y) + z$
- identity: $x + 0 = 0 + x = x$
- inverses: $x + (-x) = (-x) + x = 0$

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Who uses monoids??  Turns out, you do!

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Monoids as Collections

In a free monoid M, no sum cancels out.

Take $x \in M$ → $x + 0 = x$, same length

Take $x \in M$ → $x + y$, a longer sum

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- [1,3] + [3,7] = [1,3,3,7]
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A log is a typical free monoid.

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“Easy” Parallelism with Functional Programming

Higher-order functions are easier to handle.

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for(i = 0; i < n; i++)
  vect[i] = vect[i] + 2;
```

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map (λ x → x + 2) vect
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easier to parallelise

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easier to parallelise

- each step in `for` explicitly depends on the previous one: `i = i+1`
- the behaviour of `map` is explicitly fixed

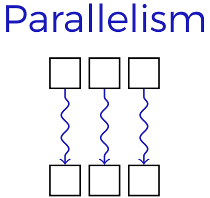
Parallelism vs. Concurrency. Statically.

Parallelism

Concurrency

Parallelism vs. Concurrency. Statically.

1. problem → independent subproblems
2. solve subproblems independently
   - no shared resources



multiple threads share resources
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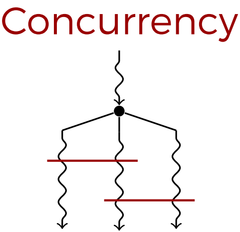

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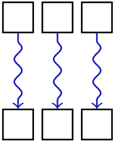

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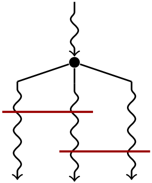

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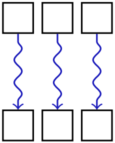

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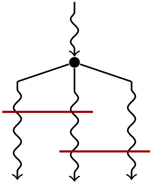

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- monads

Outline

Successive Lookups

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personByName :: String -> Maybe Person
carByPerson :: Person -> Maybe Car
model :: Car -> Maybe String
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Imagine what happens if one has longer chains.

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Factoring out Patterns (Monads! \o/)

We often want to do the same thing over and over between two function calls:

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Monads help factor out such patterns.

Conclusion

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Don't overdo it tho.

- that's the subject of my next talk