Network Medicine

Petri Nets: Extensions

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pn-biomodelling.html

Petri Nets: Definition

$$N = (P, T, W, M_0)$$

- $P = \{p_1, \ldots, p_n\}$: the set of places

- $T = \{t_1, \ldots, t_m\}$: the set of transitions

- $W : ((P \times T) \cup (T \times P)) \to \mathbb{N}$: the weight function
  - assigns multiplicities to arcs

- $M_0 : P \to \mathbb{N}$: the initial marking
  - the initial number of tokens in places

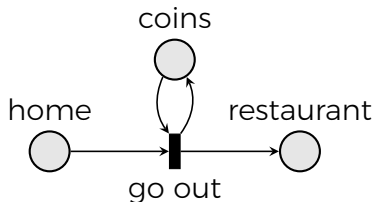Are Petri nets Turing complete?

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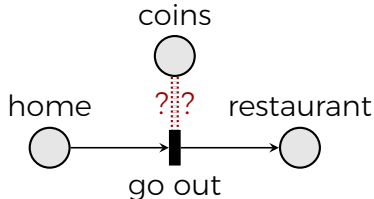Can any algorithm be implemented?

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Can any system be modelled?

Model: "I only go out if I have money."

Model: "I don't go out if I have money."

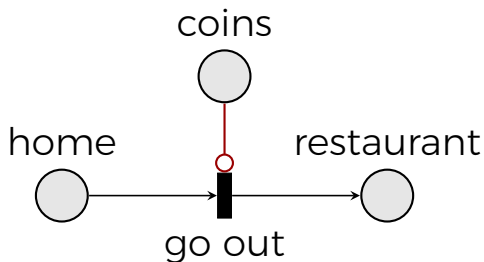# Petri Nets and Turing Completeness

Standard Petri nets are not Turing complete.

— not everything can be implemented/modelled

$+$ automatic analysis is possible

   ▶ Turing-complete systems are difficult to analyse (too expressive)
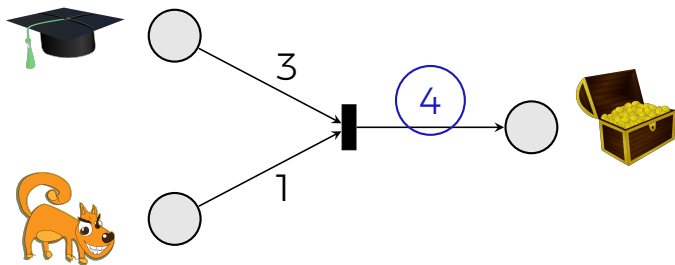
Petri Nets with Inhibitor Arcs

"I don't go out if I have money."

coins

home   restaurant

go out

"go out" is inhibited by presence of tokens in "coins".

Petri nets with inhibitor arcs are Turing complete.

Token Discrimination

https://openclipart.org/
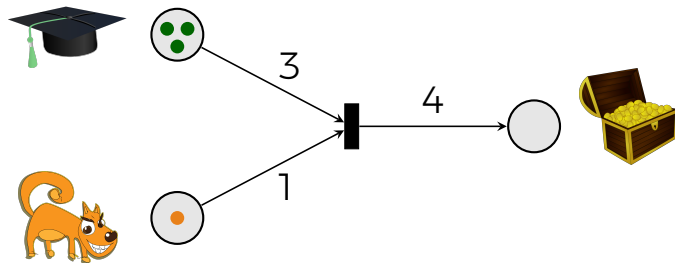
Token Discrimination

3 students + 1 dog
find the treasure!
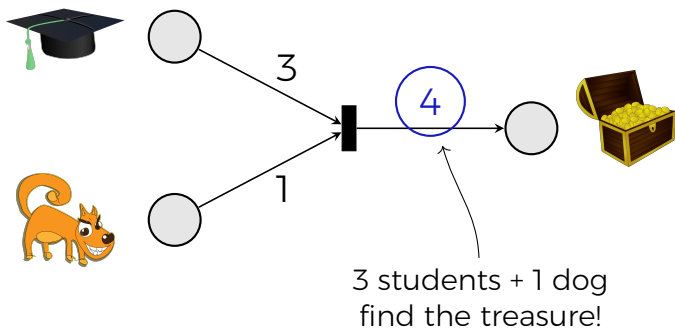
How to differentiate between the tokens?

Coloured Petri Nets CPN

Tokens may be distinguished by colours.

Colours are assigned arbitrarily.

▶ no relationship with places is enforced

Coloured Petri Nets   CPN

Tokens may be distinguished by colours.

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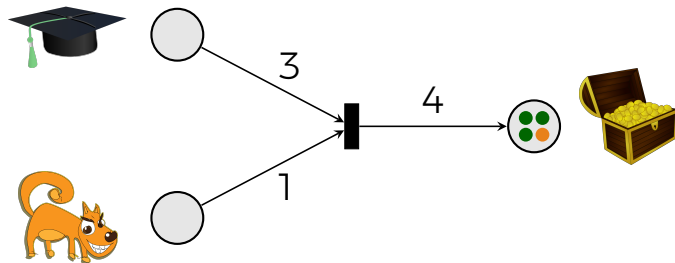
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Coloured Petri Nets CPN

Tokens may be distinguished by colours.

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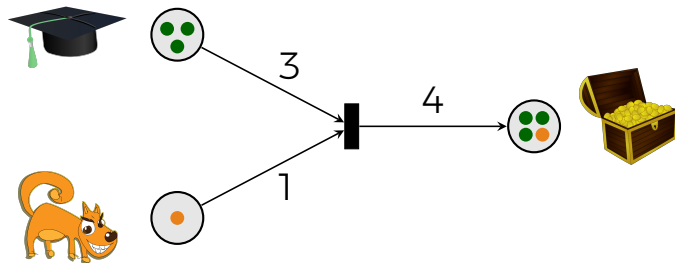# Coloured Petri Nets and Turing Completeness

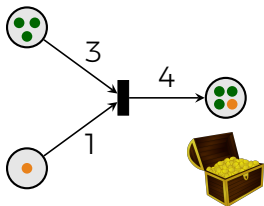Are coloured Petri nets Turing complete?
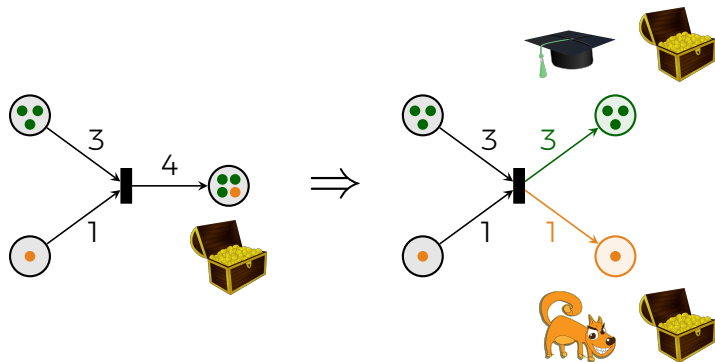
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Can any algorithm be implemented?

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Can any system be modelled?

Simple Coloured Petri Nets: Not Turing Complete

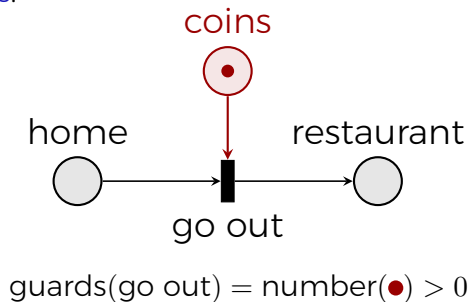# Simple Coloured Petri Nets: Not Turing Complete

Coloured tokens $=$ multiple places

https://openclipart.org/

Coloured Petri Nets with Guards

Transitions in coloured Petri nets are typically equipped with guards.

coins

home   restaurant

go out

$guards(go\ out) = number(\bullet) > 0$

Coloured Petri nets with guards are Turing complete

+ expressive modelling
− difficult analysis
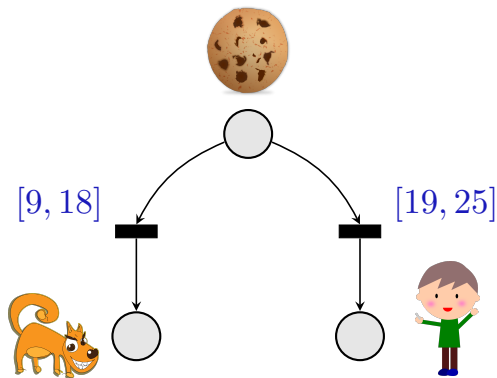
Concurrency in Time

The dog may consume the cookie between 9am and 6pm.

I may consume the cookie between 7pm and 1am.

Timed Petri Nets

A time interval $[a_t, b_t]$ is associated with each transition t.

- ► t must fire after at least a_t time units
- ► t must fire before at most b_t time units
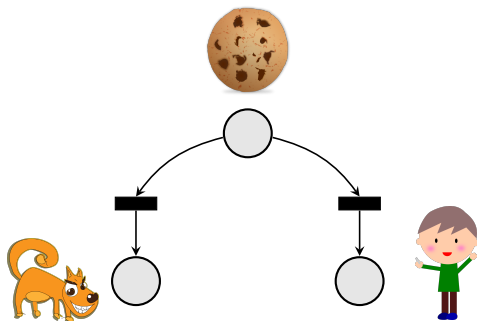
Randomised Concurrency

~~The dog may consume the cookie between~~ ~~9am~~ ~~and~~ ~~6pm~~.

~~I may consume the cookie between~~ ~~7pm~~ ~~and~~ ~~1am~~.

We just grab the cookie whenever we want!

Stochastic Petri Nets

A random variable X_t is associated with each transition t.

The firing delay of t is given by X_t.

+ easier to analyse   continuous-time Markov chains

— less modelling expressiveness for time

Cookie Metabolism

Model the following:

- ▶ I have 1.5 cookies left.

- ▶ Eating 0.3 of a cookie makes me feel 20% better.

- ▶ I eat 0.5 cookies per hour.

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- ▶ I have 1.5 cookies left.

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Conventional Petri nets are discrete—no fractional quantities or fractional consumption rates.

Continuous Petri Nets

- Places store continuous values
- Transitions have continuous firing rates
- Similar to ODE models
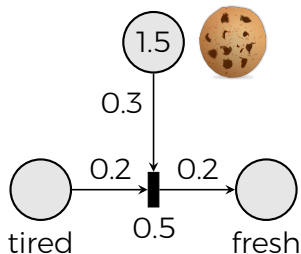
Convenient graphical notation for metabolic pathways.

Essentially a different formalism.

Summary

We have briefly discussed:

- ▶ Petri nets with inhibitor arcs
- ▶ coloured Petri nets   CPN
- ▶ timed Petri nets
- ▶ stochastic Petri nets
- ▶ continuous Petri nets

<div align="center">

There are more!   We won't go there.

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📖 Baldan, P., Cocco, N., Marin, A. et al. "Petri nets for modelling metabolic pathways: a survey" Nat Comput (2010) 9: 955. https://doi.org/10.1007/s11047-010-9180-6