



Network Medicine

Petri Nets: Definitions

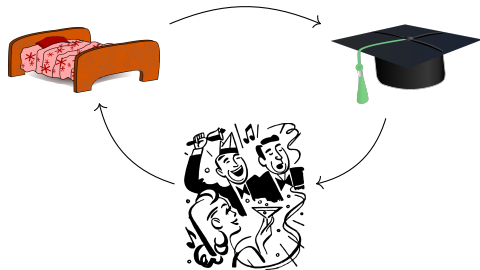
Sergiu Ivanov

sergiu.ivanov@univ-evry.fr

[https://www.ibisc.univ-evry.fr/~sivanov/en/
pn-biomodelling.html](https://www.ibisc.univ-evry.fr/~sivanov/en/pn-biomodelling.html)

Abstractly Moving Around

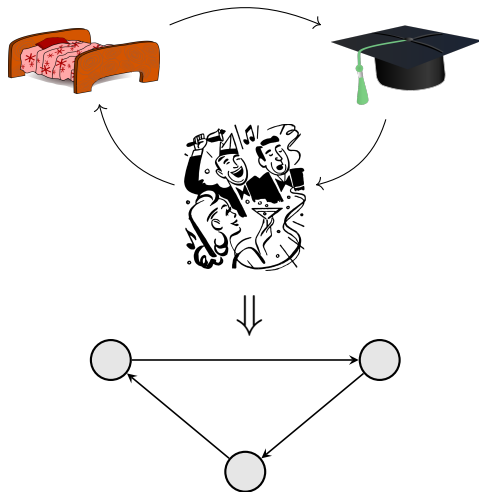
How to represent **similar entities** moving around?



<https://openclipart.org/>

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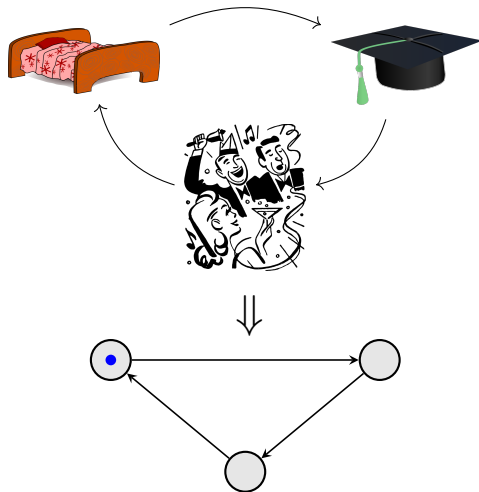
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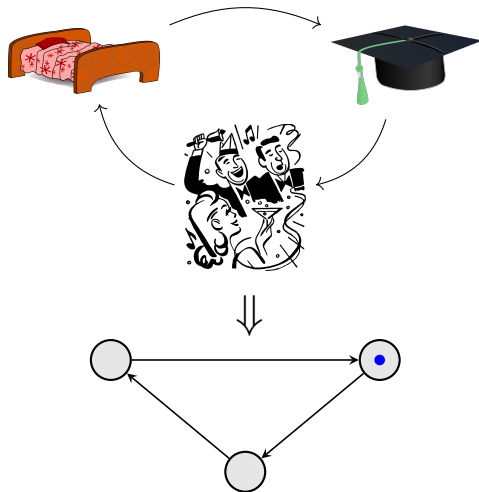
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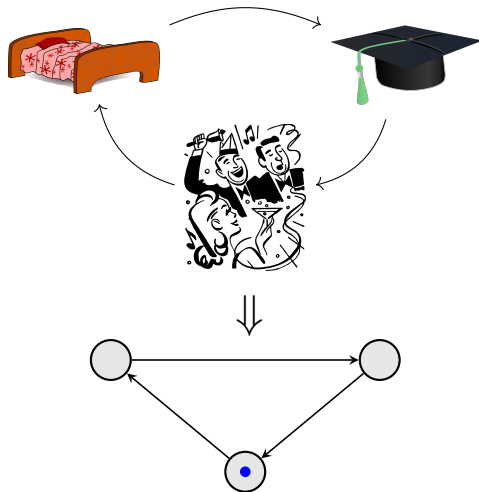
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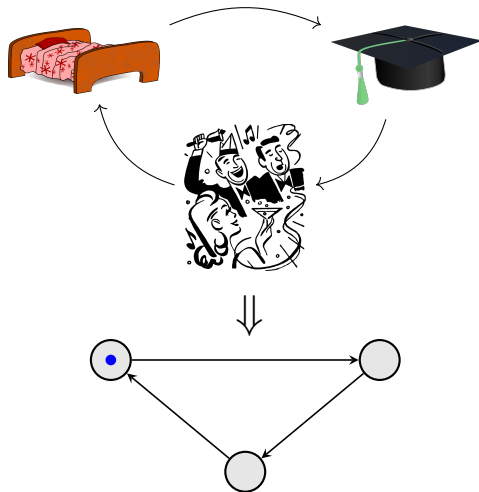
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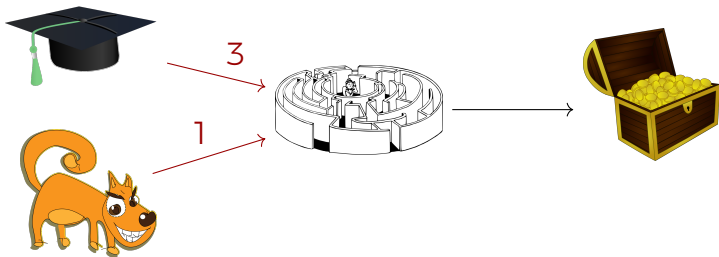
Abstractly Moving Around

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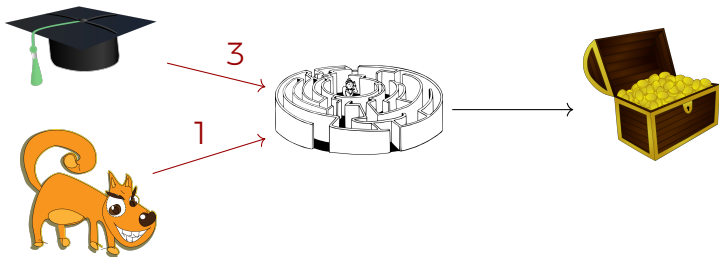
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Abstractly Moving Around Together

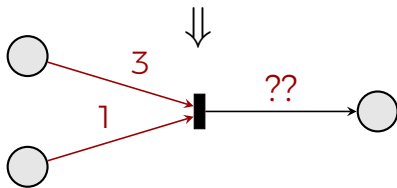


One dog may guide a group of 3 students.

Abstractly Moving Around Together



One dog may guide a group of 3 students.



Conservation Laws?

We are building an **abstract model** of entities moving around and interacting.

Do we need to **impose conservation**?

Conservation Laws?

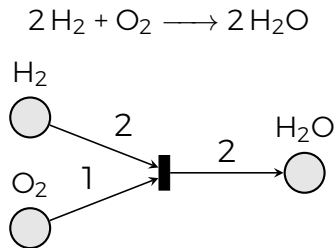
We are building an **abstract model** of entities moving around and interacting.

Do we need to **impose conservation**?

We can **later** require it for specific models.

- ▶ not in the general case

The numbers do not add up.



Petri Nets: Historical Note

Invented in August 1939 by Carl Adam Petri—at the age of 13—for describing **chemical processes**.

A **graphical notation** for stepwise processes that include choice, iteration, and concurrent execution.

- ▶ discrete dynamical systems

Have a **well developed** theory.



Carl Adam Petri

<https://ww2.informatik.hu-berlin.de/top/lehre/petriweb/>
https://en.wikipedia.org/wiki/Petri_net

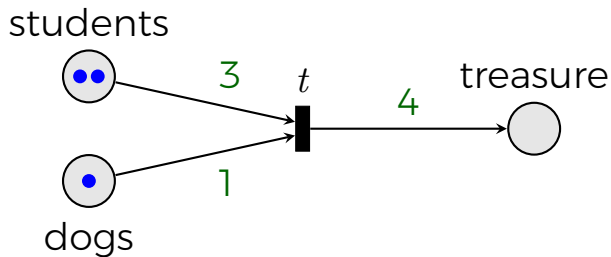
Petri Nets: Definition

$$N = (P, T, W, M_0)$$

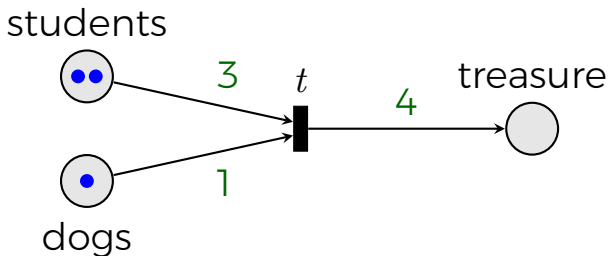
- ▶ $P = \{p_1, \dots, p_n\}$: the set of places
- ▶ $T = \{t_1, \dots, t_m\}$: the set of transitions
- ▶ $W : ((P \times T) \cup (T \times P)) \rightarrow \mathbb{N}$: the weight function
 - ▶ assigns multiplicities to arcs
- ▶ $M_0 : P \rightarrow \mathbb{N}$: the initial marking
 - ▶ the initial number of tokens in places



Illustrated Definition



Illustrated Definition



$$P = \{\text{students, dogs, treasure}\} \quad T = \{t\}$$

	(students, t)	(dogs, t)	(t, treasure)
W	3	1	4

	students	dogs	treasure
M_0	2	1	0

Basic Dynamics

How does this net *evolve*?

students



3

t

4

treasure



1



dogs

	students	dogs	treasure
M_0	2	1	0

Basic Dynamics

How does this net *evolve*?

students



3

t

treasure

4



1

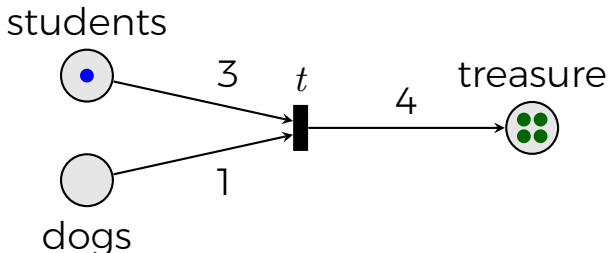
dogs



	students	dogs	treasure	Marking space	
M_0	4	1	0	$(4, 1, 0)$	$s^4 d^1 t^0$

Basic Dynamics

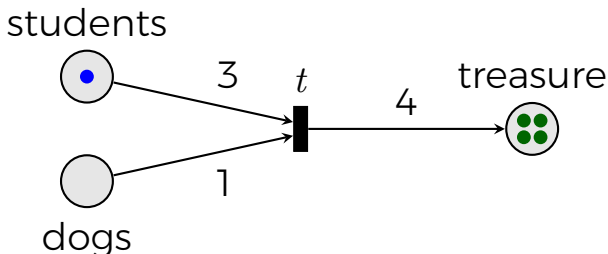
How does this net *evolve*?



	students	dogs	treasure	Marking space	
M_0	4	1	0	$(4, 1, 0)$	$s^4 d^1 t^0$
M_1	1	0	4	$(1, 0, 4)$	$s^1 d^0 t^4$

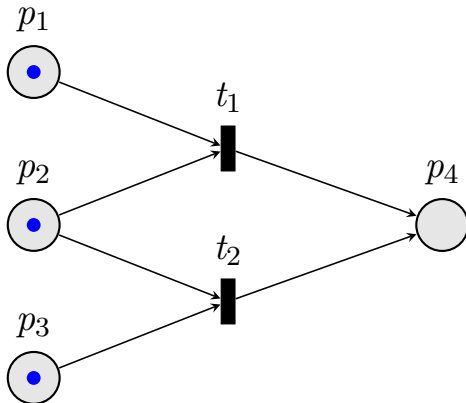
Basic Dynamics

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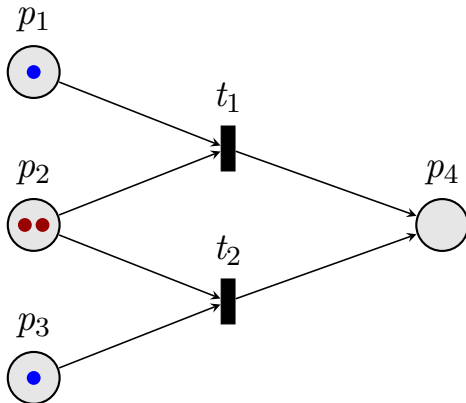


	students	dogs	treasure	Marking space	
M_0	4	1	0	$(4, 1, 0)$	$s^4 d$
M_1	1	0	4	$(1, 0, 4)$	st^4

How does **this** net **evolve**?

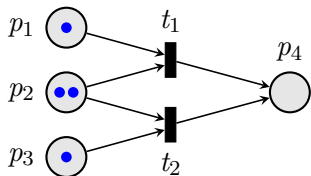


How does **this** net **evolve**?



Evolution Modes

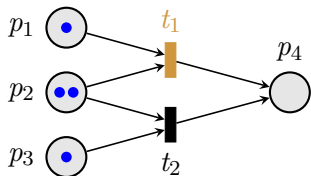
Asynchronous (sequential)



- ▶ transitions fire one by one
- ▶ **arbitrary** choice
 - ▶ non-deterministic

Evolution Modes

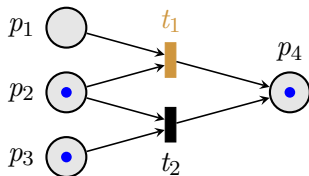
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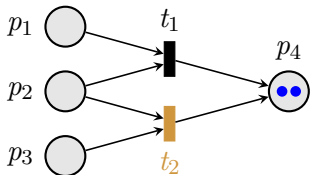
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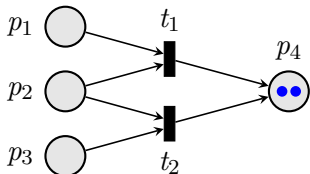
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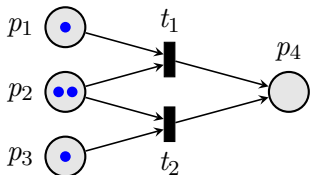
Evolution Modes

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Synchronous (parallel)

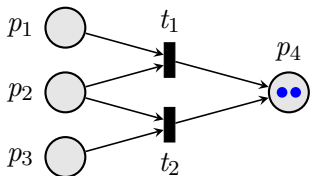


- ▶ all enabled transitions fire
 - ▶ in one single step

also more exotic modes

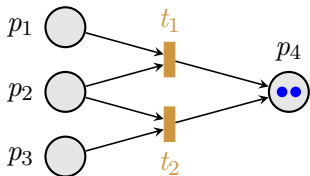
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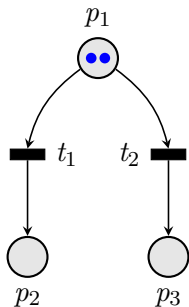


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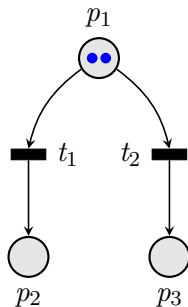
Asynchronous vs. Synchronous

Asynchronous



Transitions:

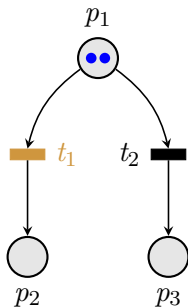
Synchronous



Transitions:

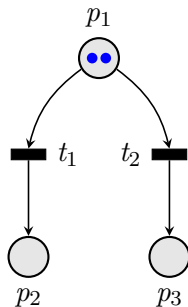
Asynchronous vs. Synchronous

Asynchronous



Transitions: t_1

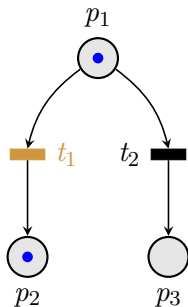
Synchronous



Transitions:

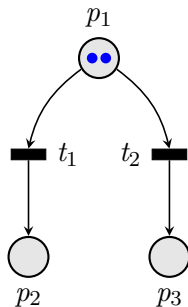
Asynchronous vs. Synchronous

Asynchronous



Transitions: t_1

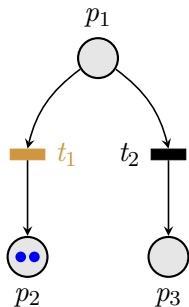
Synchronous



Transitions:

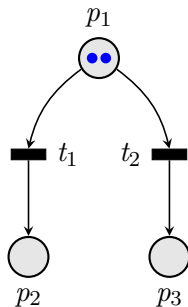
Asynchronous vs. Synchronous

Asynchronous



Transitions: t_1 t_1

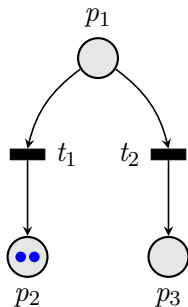
Synchronous



Transitions:

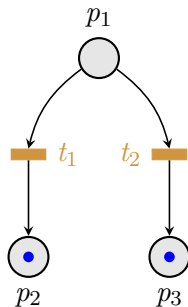
Asynchronous vs. Synchronous

Asynchronous



Transitions: $t_1 t_1$

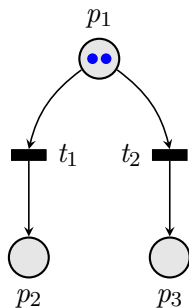
Synchronous



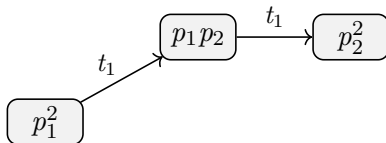
Transitions: t_1
 t_2

The marking p_2^2 is unreachable in synchronous mode.

Asyn vs. Syn: State Graphs

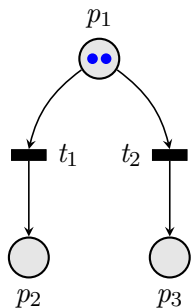


Asynchronous

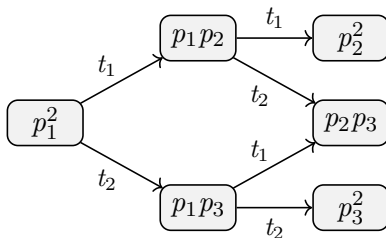


Asynchronous evolution is often **non-deterministic**.

Asyn vs. Syn: State Graphs

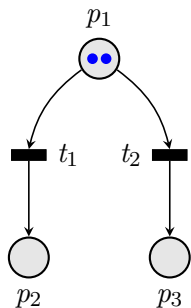


Asynchronous

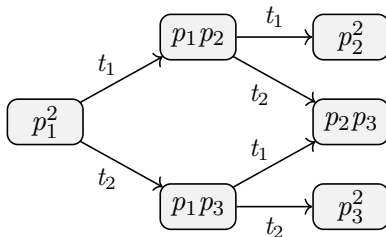


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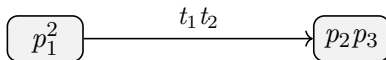
Asyn vs. Syn: State Graphs



Asynchronous



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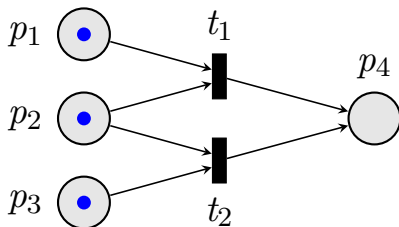
Asynchronous evolution is often **non-deterministic**.

Side Note: Non-determinism with Syn?

Are synchronous nets **always deterministic**?

Side Note: Non-determinism with Syn?

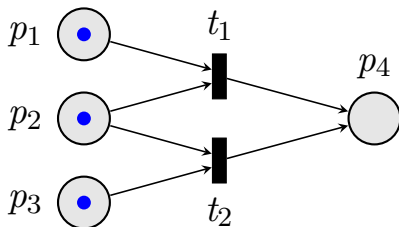
Are synchronous nets **always deterministic**?



Answer: No.

Side Note: Non-determinism with Syn?

Are synchronous nets **always deterministic**?



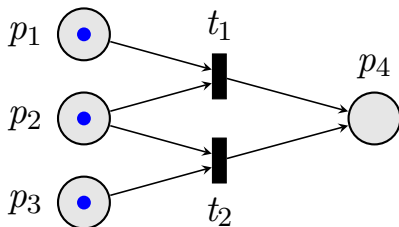
Answer: No.

Bonus: Do we always reach the **same markings**?

▶ confluency

Side Note: Non-determinism with Syn?

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Answer: No.

Bonus: Do we always reach the **same markings**?

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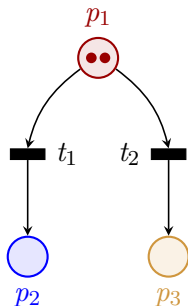
Answer: Nope.

Petri Nets as Multiset Rewriting

A **multiset** is a set with repetitions.

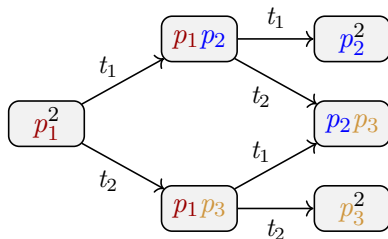
- ▶ $\{a, a, b\}$ and $\{a, b\}$ are different multisets
- ▶ $\{a, a, b\}$ and $\{a, b, a\}$ are the same multiset

Petri net transitions = multiset rewriting rules



$t_1 \mapsto p_1 \rightarrow p_2$

$t_2 \mapsto p_1 \rightarrow p_3$



<https://en.wikipedia.org/wiki/Multiset>