Network Medicine

Petri Nets: Definitions

Sergiu Ivanov

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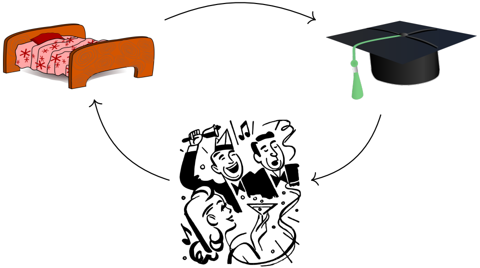
http://lacl.fr/~sivanov/doku.php?id=en:pn-biomodelling

Abstractly Moving Around

How to represent similar entities moving around?



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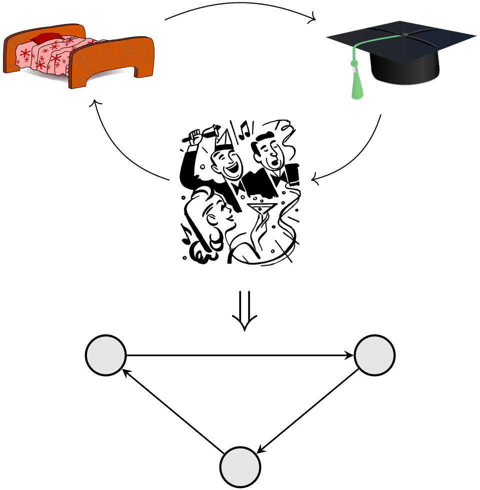

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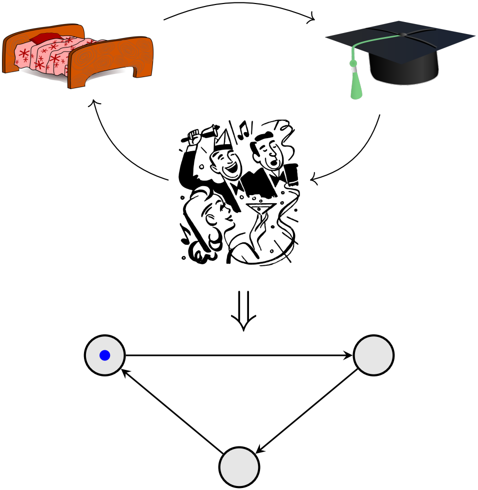

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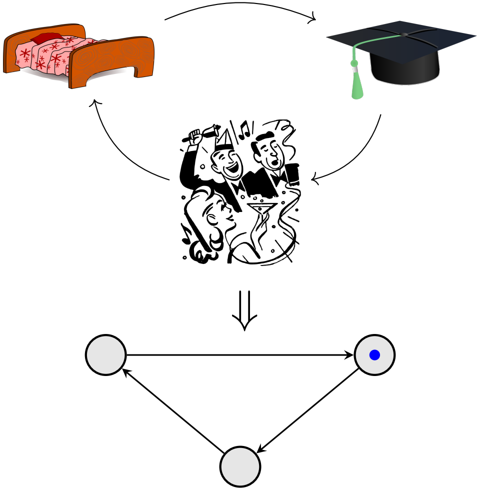

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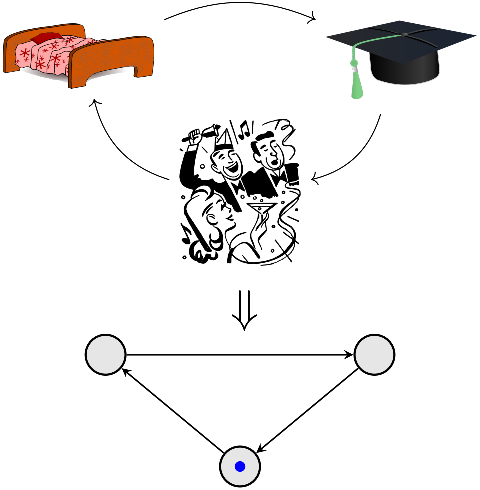

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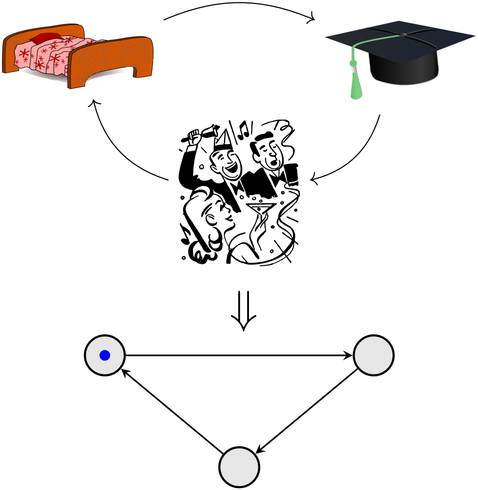

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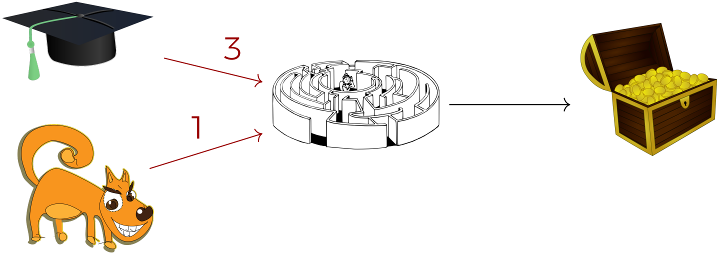
Abstractly Moving Around Together



One dog may guide a group of 3 students.

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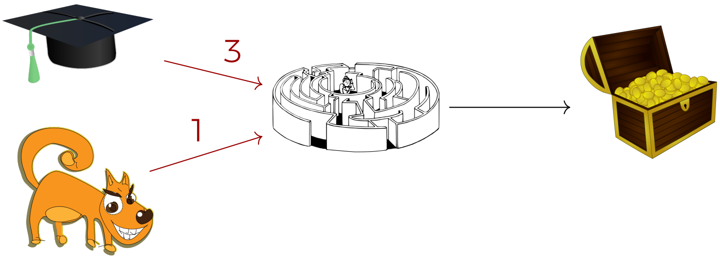

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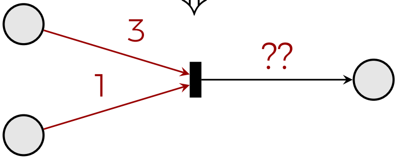
⇓



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Conservation Laws?

We are building an abstract model of entities moving around and interacting.

Do we need to impose conservation?

Conservation Laws?

We are building an abstract model of entities moving around and interacting.

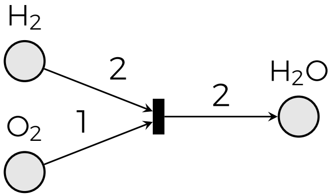
Do we need to impose conservation?

We can later require it for specific models.

- not in the general case

The numbers do not add up.

$$2\,H_2 + O_2 \longrightarrow 2\,H_2O$$



Petri Nets: Historical Note

Invented in August 1939 by Carl Adam Petri—at the age of 13—for describing chemical processes.

A graphical notation for stepwise processes that include choice, iteration, and concurrent execution.

- discrete dynamical systems

Have a well developed theory.

Carl Adam Petri

https://www2.informatik.hu-berlin.de/top/lehre/petriweb/
https://en.wikipedia.org/wiki/Petri_net

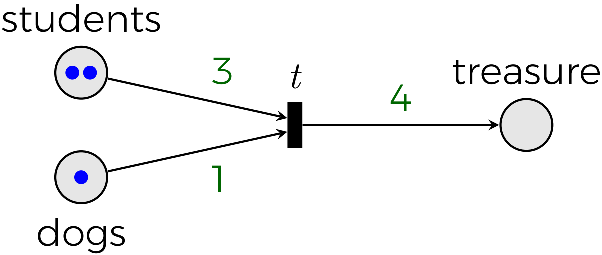
Petri Nets: Definition

$$N = (P, T, W, M_0)$$

- $P = \{p_1, \ldots, p_n\}$: the set of places 
- $T = \{t_1, \ldots, t_m\}$: the set of transitions 
- $W : ((P \times T) \cup (T \times P)) \to \mathbb{N}$: the weight function
  - assigns multiplicities to arcs 

- $M_0 : P \to \mathbb{N}$: the initial marking
  - the initial number of tokens in places 

Illustrated Definition



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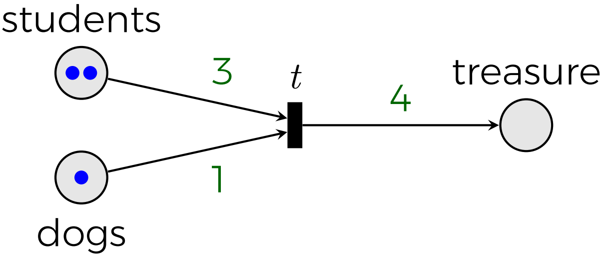


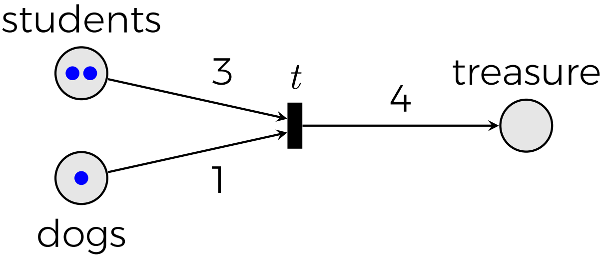
$P = \{\text{students}, \text{dogs}, \text{treasure}\}$ $\qquad T = \{t\}$

	(students, t)	(dogs, t)	(t, treasure)
W	3	1	4

	students	dogs	treasure
M_0	2	1	0

Basic Dynamics

How does this net evolve?



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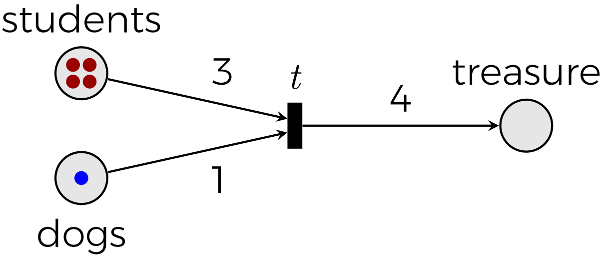


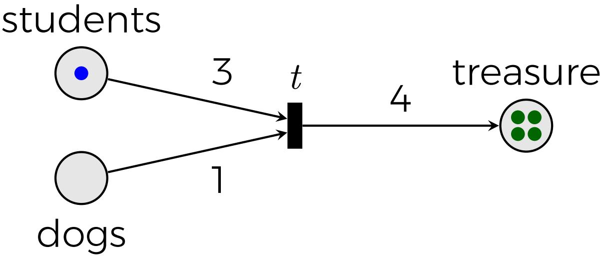
	students	dogs	treasure
M_0	4	1	0

Marking space

$(4, 1, 0)$ $s^4 d^1 t^0$

Basic Dynamics

How does this net evolve?



	students	dogs	treasure
M_0	4	1	0
M_1	1	0	4

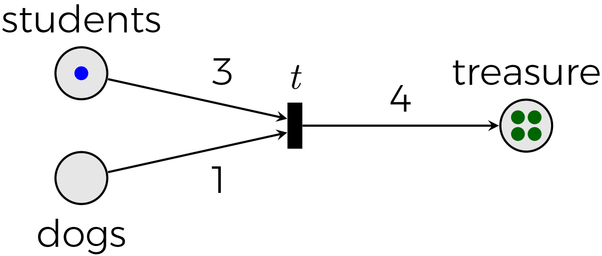
Marking space

$(4,1,0) \to (1,0,4)$

$s^4 d^1 t^0 \to s^1 d^0 t^4$

Basic Dynamics

How does this net **evolve**?



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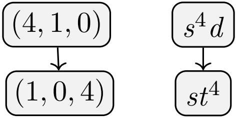

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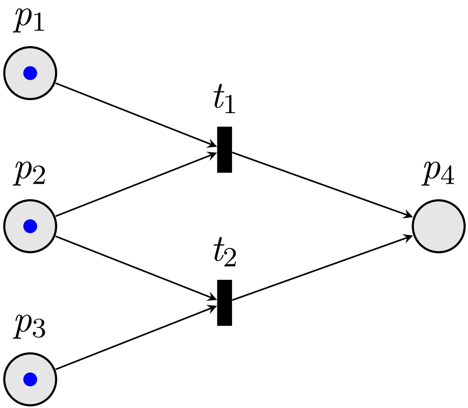

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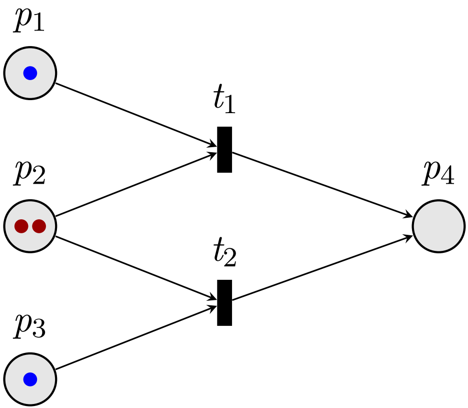
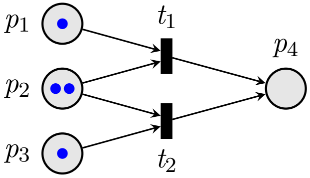


Evolution Modes

Asynchronous (sequential)



- transitions fire one by one
- arbitrary choice
  - non-deterministic

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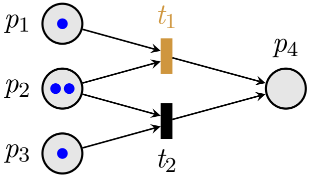

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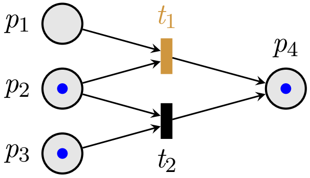

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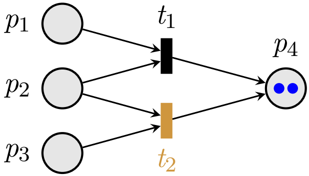

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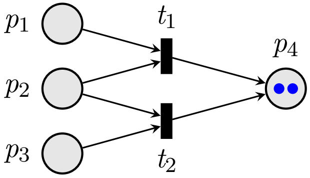

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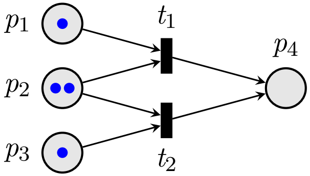
Synchronous (parallel)



- all enabled transitions fire
  - in one single step

also more exotic modes

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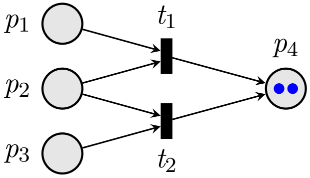

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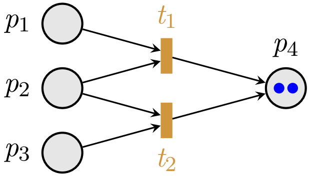

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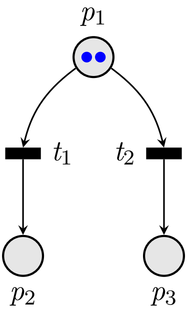
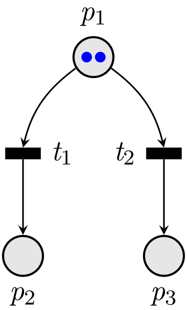
Asynchronous vs. Synchronous

Asynchronous



Transitions:

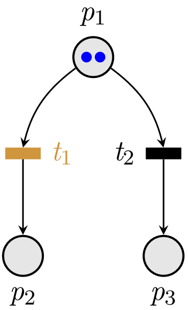
Synchronous



Transitions:

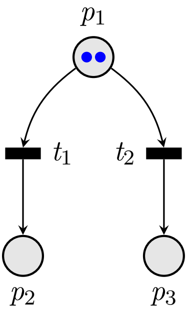
Asynchronous vs. Synchronous

Asynchronous



Transitions: t_1

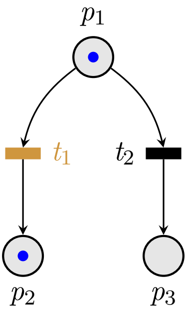
Synchronous



Transitions:

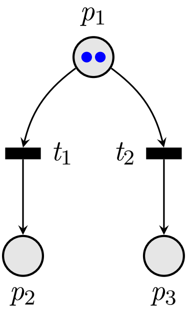
Asynchronous vs. Synchronous

Asynchronous



Transitions: t_1

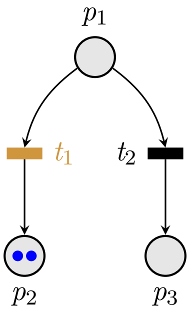
Synchronous



Transitions:

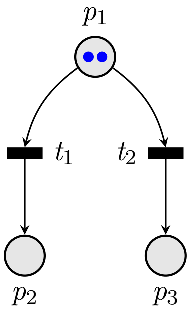
Asynchronous vs. Synchronous

Asynchronous



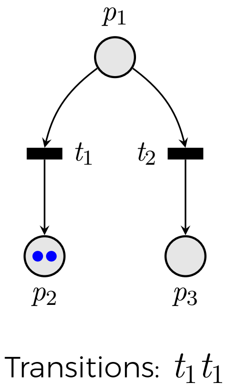
Transitions: $t_1 \, t_1$

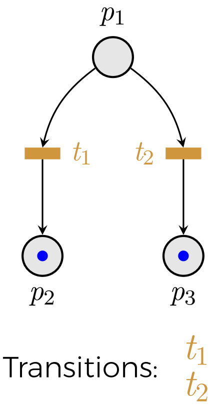
Synchronous



Transitions:

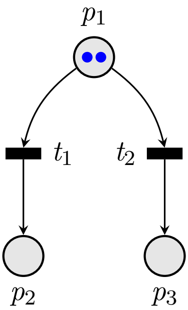
Asynchronous vs. Synchronous





The marking p_2^2 is unreachable in synchronous mode.

Asyn vs. Syn: State Graphs



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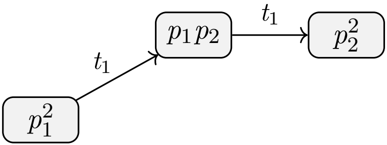

Asynchronous evolution is often non-deterministic.

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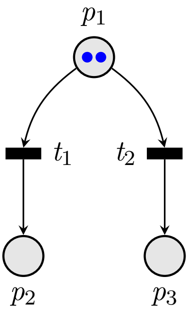

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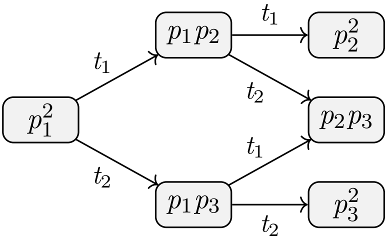

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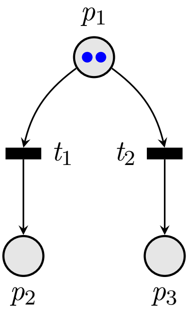

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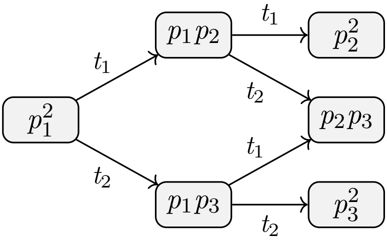

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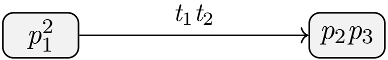

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Side Note: Non-determinism with Syn?

Are synchronous nets always deterministic?

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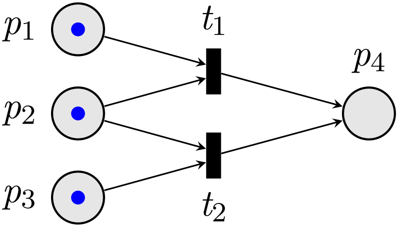
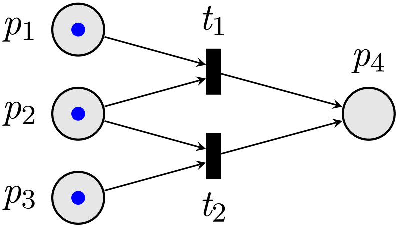


Answer: No.

Side Note: Non-determinism with Syn?

Are synchronous nets always deterministic?



Answer: No.

Bonus: Do we always reach the same markings?

- confluency

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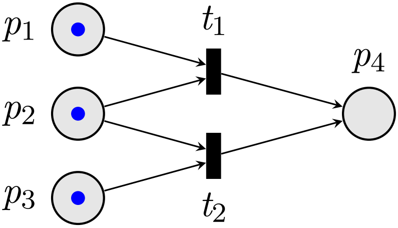


Answer: No.

Bonus: Do we always reach the same markings?

- confluency

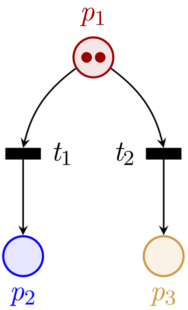
Answer: Nope.

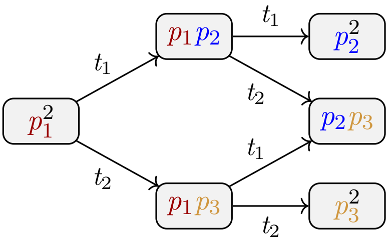
Petri Nets as Multiset Rewriting

A multiset is a set with repetitions.

- $\{a, a, b\}$ and $\{a, b\}$ are different multisets
- $\{a, a, b\}$ and $\{a, b, a\}$ are the same multiset

Petri net transitions = multiset rewriting rules





$$t_1 \mapsto p_1 \to p_2$$
$$t_2 \mapsto p_1 \to p_3$$

https://en.wikipedia.org/wiki/Multiset